ONLINE APPENDIX

Proof of Lemma 1:

First, note that the consumer's expected payoff of adopting the alternative in the search mode is non-positive if the purchasing threshold $\overline{x} \leq 0$. In contrast, because there is a positive probability of reaching any belief within a given period of time, the consumer's expected payoff of adopting the alternative in the search mode is strictly positive if the purchasing threshold $\overline{x} > 0$. Therefore, the optimal $\overline{x} > 0$.

Suppose the purchasing threshold in the no-search mode is higher than that in the search mode, $\widetilde{x} \geq \overline{x}$. Because the belief evolves continuously, one can see that the consumer never adopts the alternative in the no-search mode. We now show that the consumer can be strictly better off by using a lower purchasing threshold in the no-search region. Consider $\widetilde{x}' := \overline{x} - \epsilon$, where $\epsilon > 0$. Consider $x = \widetilde{x}'$ in the no-search mode. The consumer will adopt the alternative immediately and obtain a payoff of $\widetilde{x}'$ if the threshold in the no-search mode is $\tilde{x}'$.

Denote the consumer's expected payoff in the no-search mode by $W(x)$ and in the search mode by $V(x)$ if the threshold in the no-search mode is $\widetilde{x}$. We have $W(\widetilde{x}') = \mathbb{E}[e^{-rN_1}V(\widetilde{x}')]$, where N_1 is the time of the first arrival of a Poisson process with rate β. We have:

$$\begin{aligned}
W(\widetilde{x}') =& \mathbb{E}[e^{-rN_1}V(\widetilde{x}')] \\
\leq& \mathbb{E}[e^{-rN_1}V(\overline{x})] \\
=& \mathbb{E}[e^{-rN_1}]\overline{x} \\
<& [\mathbb{P}(N_1 \leq 1) \cdot 1 + \mathbb{P}(N_1 > 1) \cdot e^{-r}]\overline{x} \\
=& [(1 - e^{-\beta}) \cdot 1 + e^{-\beta} \cdot e^{-r}]\overline{x} \\
=& \overline{x} - e^{-\beta}(1 - e^{-r})\overline{x}
\end{aligned}$$

One can then see that $W(\widetilde{x}') < \widetilde{x}' = \overline{x} - \epsilon$ for ϵ small enough. Therefore, the threshold $\widetilde{x} \geq \overline{x}$ cannot be optimal.

Derivation of Solution to Base Case with Discounting:

Given that $\lim_{x \to -\infty} V(x) = 0$, as the expected payoff of the DM has to approach zero if the expected payoff of the alternative approaches negative infinity, we have that the solution to (5) satisfies

$$V(x) = A_1 e^{\eta x} \tag{i}$$

where A_1 is a constant to be determined.

Similarly, applying Itô's Lemma to (2), we can obtain the solution to the second order differential equation in $V(x)$ for $x \in (\widetilde{x}, \overline{x})$ as

$$V(x) = A_2 e^{\widetilde{\eta} x} + A_3 e^{-\widetilde{\eta} x} + \frac{\lambda}{r+\lambda} x, \tag{ii}$$

where A_2 and A_3 are constants to be determined.

Using value matching and smooth pasting of $V(x)$ at $\widetilde{x}$ and $\overline{x}$, $V(\widetilde{x}^-) = V(\widetilde{x}^+), V'(\widetilde{x}^-) = V'(\widetilde{x}^+)$, $V(\overline{x}) = \overline{x}$, and $V'(\overline{x}) = 1$, and $W(\widetilde{x}) = \widetilde{x}$, we obtain the following system of five equations to obtain $\widetilde{x}, \overline{x}, A_1, A_2$, and A_3.

$$\begin{aligned}
A_2 e^{\widetilde{\eta}\overline{x}} + A_3 e^{-\widetilde{\eta}\overline{x}} + \frac{\lambda}{r+\lambda}\overline{x} &= \overline{x} && \text{(iii)}\\
\widetilde{\eta} A_2 e^{\widetilde{\eta}\overline{x}} - \widetilde{\eta} A_3 e^{-\widetilde{\eta}\overline{x}} + \frac{\lambda}{r+\lambda} &= 1 && \text{(iv)}\\
A_2 e^{\widetilde{\eta}\widetilde{x}} + A_3 e^{-\widetilde{\eta}\widetilde{x}} + \frac{\lambda}{r+\lambda}\widetilde{x} &= A_1 e^{\eta\widetilde{x}} && \text{(v)}\\
\widetilde{\eta} A_2 e^{\widetilde{\eta}\widetilde{x}} - \widetilde{\eta} A_3 e^{-\widetilde{\eta}\widetilde{x}} + \frac{\lambda}{r+\lambda} &= \eta A_1 e^{\eta\widetilde{x}} && \text{(vi)}\\
\frac{\beta}{r+\beta} A_1 e^{\eta\widetilde{x}} &= \widetilde{x}. && \text{(vii)}
\end{aligned}$$

Using (iii)-(vii), we can obtain a system of two equations to obtain $\widetilde{x}$ and $\overline{x}$ as

$$\begin{aligned}
e^{\widetilde{\eta}(\overline{x}-\widetilde{x})} &= \frac{\frac{r}{r+\lambda}\overline{x} + \frac{r}{\widetilde{\eta}(r+\lambda)}}{\widetilde{x}\left(\frac{r+\beta}{\beta} - \frac{\lambda}{r+\lambda}\right) + \frac{1}{\widetilde{\eta}}\left(\eta\widetilde{x}\frac{r+\beta}{\beta} - \frac{\lambda}{r+\lambda}\right)} && \text{(viii)}\\
e^{\widetilde{\eta}(\overline{x}-\widetilde{x})} &= \frac{\widetilde{x}\left(\frac{r+\beta}{\beta} - \frac{\lambda}{r+\lambda}\right) - \frac{1}{\widetilde{\eta}}\left(\eta\widetilde{x}\frac{r+\beta}{\beta} - \frac{\lambda}{r+\lambda}\right)}{\frac{r}{r+\lambda}\overline{x} - \frac{r}{\widetilde{\eta}(r+\lambda)}}. && \text{(ix)}
\end{aligned}$$

Using $\delta = \overline{x} - \widetilde{x}$ we can rewrite (viii) and (ix), as a system of equations for δ and $\widetilde{x}$ as

$$\begin{aligned}
\widetilde{x} &= \beta\frac{r + r\widetilde{\eta}\delta + \lambda D}{D[\widetilde{\eta}r(r+\beta+\lambda) + \eta(r+\beta)(r+\lambda)] - \widetilde{\eta}\beta r} && \text{(x)}\\
\widetilde{x} &= \beta\frac{\lambda + rD - \widetilde{\eta}r\delta D}{\widetilde{\eta}r\beta D + \eta(r+\beta)(r+\lambda) - \widetilde{\eta}r(r+\beta+\lambda)} && \text{(xi)}
\end{aligned}$$

where $D = e^{\widetilde{\eta}\delta} = e^{\widetilde{\eta}(\overline{x}-\widetilde{x})}$.[20] Using (x) and (xi) we can obtain (6) in the main text, from which we can obtain δ. We can then use (x) or (xi) to obtain $\widetilde{x}$. Lastly, we can obtain $\overline{x}$ from δ and $\widetilde{x}$ because $\overline{x} = \widetilde{x} + \delta$.

Derivation of equation (10) in the Base Case: Since $\delta \to 0$ as $\beta \to +\infty$ and $D = 1 + \widetilde{\eta}\delta + o(\delta)$,

[20] Since D depends on $\overline{x}$, (x) and (xi) can also be viewed as a system of equations for $\overline{x}$ and $\widetilde{x}$.

we have

$$
\begin{aligned}
&\beta(D-1)^2 \to 2(r+\lambda) \\
\Rightarrow\ &\beta[\widetilde{\eta}\delta + o(\delta)]^2 = 2(r+\lambda) + o(1) \\
\Rightarrow\ &\beta\widetilde{\eta}^2\delta^2 = 2(r+\lambda) + o(1) \\
\Rightarrow\ &\beta\delta^2 = \sigma^2 + o(1) \\
\Rightarrow\ &\sqrt{\beta}\delta \to \sigma, \text{ as } \beta \to +\infty
\end{aligned}
$$

Proof of Proposition 1:

We provide proof for the comparative statics w.r.t. σ^2. The proofs for the comparative statics w.r.t. β, r, and λ are similar.

Given $\sigma_s^2 < \sigma_\ell^2$ and the corresponding cutoff beliefs $(\overline{x}_s, \widetilde{x}_s)$, $(\overline{x}_\ell, \widetilde{x}_\ell)$, we want to show that $\overline{x}_\ell \geq \overline{x}_s$ and $\widetilde{x}_\ell \geq \widetilde{x}_s$.

Suppose $\overline{x}_\ell < \overline{x}_s$. Then $V_s(\overline{x}_\ell) > \overline{x}_\ell$, because the DM keeps searching for information when the belief is $\overline{x}_\ell$ and $\sigma^2 = \sigma_s^2$. Also, $V_\ell(\overline{x}_\ell) = \overline{x}_\ell$, because the DM takes the alternative when the belief is $\overline{x}_\ell$ and $\sigma^2 = \sigma_\ell^2$. Therefore, $V_s(\overline{x}_\ell) > V_l(\bar{x}_l)$.

However, one can see that the DM can achieve a payoff of at least $V_s(\bar{x}_\ell)$ when $\sigma^2 = \sigma_\ell^2$ and the belief is $\overline{x}_\ell$ by using the optimal strategy when $\sigma^2 = \sigma_s^2$ (which may be sub-optimal when $\sigma^2 = \sigma_\ell^2$). Therefore, $V_s(\overline{x}_\ell) \leq V_\ell(\overline{x}_\ell)$, a contradiction. So, $\overline{x}_\ell \geq \overline{x}_s$.

Now suppose that $\widetilde{x}_\ell < \widetilde{x}_s$. Then $W_s(\widetilde{x}_\ell) > \widetilde{x}_\ell$, because the DM defers the choice when the belief is $\widetilde{x}_\ell$ and $\sigma^2 = \sigma_s^2$. Also, $W_\ell(\widetilde{x}_\ell) = \widetilde{x}_\ell$ because the DM takes the alternative in the no-search mode when the belief is $\widetilde{x}_\ell$ and $\sigma^2 = \sigma_\ell^2$. Therefore, $V_s(\widetilde{x}_\ell) = \frac{r+\beta}{\beta} W_s(\widetilde{x}_\ell) > \frac{r+\beta}{\beta} W_\ell(\widetilde{x}_\ell) = V_\ell(\widetilde{x}_\ell)$.

However, one can see that the DM can achieve a payoff of at least $V_s(\widetilde{x}_\ell)$ when $\sigma^2 = \sigma_\ell^2$ and the belief is $\widetilde{x}_\ell$ by using the optimal strategy when $\sigma^2 = \sigma_s^2$ (which may be sub-optimal when $\sigma^2 = \sigma_\ell^2$). Therefore, $V_s(\widetilde{x}_\ell) \leq V_\ell(\widetilde{x}_\ell)$, a contradiction. So, $\widetilde{x}_\ell \geq \widetilde{x}_s$.

Optimal Pricing for Benchmarks:

Let V_f^{nb} denote the firm's value function in the No-Fatigue Benchmark. The consumer buys when $x > \overline{x}_{nb} + P$. Note that if $x_0 \geq \overline{x}_{nb} + P$, the consumer buys immediately, thus the firm should charge $P = x_0 - \overline{x}_{nb}$. For $x < \overline{x}_{nb} + P$, we have

$$V_f^{nb}(x) = e^{-rdt} EV(x + dx) \tag{xii}$$

Applying Itô's Lemma and solving the resulting differential equation, we get

$$V_f^{nb}(x) = A_{nb}e^{\sqrt{\frac{2r}{\sigma}}x} + B_{nb}e^{-\sqrt{\frac{2r}{\sigma}}x} \tag{xiii}$$

where A_{nb} and B_{nb} are constants to be solved. Because $V_f^{nb}(x) \to 0$ as $x \to -\infty$, we must have $B_{nb} = 0$. The constant A_{nb} is solved by applying the boundary condition $V_f^{nb}(\overline{x}_{nb} + P) = P$. This produces

$$V_f^{nb}(x) = Pe^{\sqrt{\frac{2r}{\sigma}}(x-\overline{x}_{nb}-P)} \tag{xiv}$$

which is maximized at $P_{nb}^* = \overline{x}_{nb} = \sqrt{\frac{\sigma^2}{2r}}$. The optimal price is $P_{nb}^* = \sqrt{\frac{\sigma^2}{2r}}$ for $x_0 < 2\sqrt{\frac{\sigma^2}{2r}}$ and $P_{nb}^* = x_0 - \sqrt{\frac{\sigma^2}{2r}}$ for $x_0 \geq 2\sqrt{\frac{\sigma^2}{2r}}$.

A similar analysis of the Model-Free-Fatigue Benchmark shows that the optimal price is $P_{mb}^* = \sqrt{\frac{\sigma^2}{2r}\frac{\beta}{\lambda+\beta}}$ for $x_0 < 2\sqrt{\frac{\sigma^2}{2r}\frac{\beta}{\lambda+\beta}}$ and $P_{mb}^* = x_0 - \sqrt{\frac{\sigma^2}{2r}\frac{\beta}{\lambda+\beta}}$ for $x_0 \geq 2\sqrt{\frac{\sigma^2}{2r}\frac{\beta}{\lambda+\beta}}$.

Some Analysis of Optimal Pricing:

Substituting $W_f(x) = \frac{\beta}{r+\beta}V_f(x)$ into (11), and using Itô's Lemma, we can obtain the second order differential equation in $V_f(x)$ for $x < \widetilde{x} + P$ as

$$r\frac{r+\beta+\lambda}{r+\beta}V_f(x) = \frac{\sigma^2}{2}V_f''(x). \tag{xv}$$

Given that $\lim_{x\to-\infty} V_f(x) = 0$, as the expected payoff of the firm has to approach zero if the expected payoff of the alternative approaches negative infinity, we have that the solution to (xv) satisfies

$$V_f(x) = \widetilde{A}_1 e^{\eta x} \tag{xvi}$$

where $\widetilde{A}_1$ is a constant to be determined.[21]

Similarly, applying Itô's Lemma to (12), we can solve the resulting second order differential equation in $V_f(x)$ for $x \in (\widetilde{x} + P, \overline{x} + P)$ as

$$V_f(x) = \widetilde{A}_2 e^{\widetilde{\eta}x} + \widetilde{A}_3 e^{-\widetilde{\eta}x} + \frac{\lambda}{r+\lambda}P \tag{xvii}$$

where $\widetilde{A}_2$ and $\widetilde{A}_3$ are constants to be determined.

[21]Recall that $\eta = \sqrt{\frac{2r}{\sigma^2}\frac{r+\beta+\lambda}{r+\beta}}$ and $\widetilde{\eta} = \sqrt{\frac{2(r+\lambda)}{\sigma^2}}$.

Conditions (14)-(16) can be written as:

$$\begin{aligned} P &= \widetilde{A}_2 e^{\widetilde{\eta}(\overline{x}+P)} + \widetilde{A}_3 e^{-\widetilde{\eta}(\overline{x}+P)} + \frac{\lambda}{r+\lambda} P & \text{(xviii)} \\ \widetilde{A}_1 e^{\eta(\widetilde{x}+P)} &= \widetilde{A}_2 e^{\widetilde{\eta}(\widetilde{x}+P)} + \widetilde{A}_3 e^{-\widetilde{\eta}(\widetilde{x}+P)} + \frac{\lambda}{r+\lambda} P & \text{(xix)} \\ \widetilde{A}_1 \eta e^{\eta(\widetilde{x}+P)} &= \widetilde{A}_2 \widetilde{\eta} e^{\widetilde{\eta}(\widetilde{x}+P)} - \widetilde{A}_3 \widetilde{\eta} e^{-\widetilde{\eta}(\widetilde{x}+P)}. & \text{(xx)} \end{aligned}$$

Taking the derivative of (xvii) with respect to price and making it equal to zero, yields the optimal price for $x_0 \in [x_0^*, x_0^{**}]$. This yields an equation $h(P, x_0) = 0$ which is represented by

$$h(P, x_0) = \frac{\lambda}{r+\lambda} P^* + \widetilde{A}_2 (1 - \widetilde{\eta} P^*) e^{\widetilde{\eta} x_0} + \widetilde{A}_3 (1 + \widetilde{\eta} P^*) e^{-\widetilde{\eta} x_0} = 0, \qquad \text{(xxi)}$$

where $\widetilde{A}_2$ and $\widetilde{A}_3$ are both functions of price.

In order to obtain some more specific results we consider two particular cases.

The Case of $\beta \to 0$ for $x_0 \in [x_0^*, x_0^{**}]$:

When $\beta \to 0$, we have $\eta \to \widetilde{\eta}, \widetilde{x} \to 0$, and $e^{\eta \overline{x}}(1 - \eta \overline{x}) + \frac{\lambda}{r} = 0$ (from which we recall that $\overline{x} > 1/\eta$).

From (xvii)-(xx) we can obtain that in the limit

$$\begin{aligned} V_f(x_0) =& \frac{P}{r+\lambda} \left[\frac{r(\eta \overline{x} + 1)}{2} e^{\eta(x_0 - \overline{x} - P)} - \frac{\lambda}{2} e^{\eta(P - x_0)} + \lambda \right] \\ \text{sign}\left\{ \frac{\partial V_f(x)}{\partial P} \right\} =& \text{sign}\left\{ \lambda + (1 - \eta P) \frac{r(\eta \overline{x} + 1)}{2} e^{\eta(x_0 - \overline{x} - P)} - (1 + \eta P) \frac{\lambda}{2} e^{\eta(P - x_0)} \right\}. \end{aligned}$$

Note that in this case we have $x_0^* \to 1/\eta$. So, for $x_0 > x_0^*$, we can obtain that $\frac{\partial V_f(x_0)}{\partial P} > 0$ for $P = 1/\eta$. Furthermore, we can obtain that $\frac{\partial V_f(x_0)}{\partial P} < 0$ for $P = x_0$. So, we have that for $x_0 \in [x_0^*, x_0^{**}]$ we have that $P^* \in [1/\eta, x_0]$. If the price function is continuous at x_0^{**}, then from the definition of x_0^{**} we can also obtain for this case of $\beta \to 0$ that $x_0^{**} \to \overline{x} + \frac{r+\lambda}{r\eta^2 \overline{x}.}$.

To check whether the price function is continuous at x_0^{**}, we can check whether for x_0^{**} obtained by $h(x_0^{**} - \overline{x}, x_0^{**}) = 0$ we have that $V_f(x_0^{**}, P)$ is concave in the price P when $P = x_0^{**} - \overline{x}$. This condition yields, using (8), $\lambda/r < 2a^2 - 1$ where $a > 1$ satisfies $e^a(a-1) - 2a^2 + 1 = 0$.[22] For $\lambda/r > 2a^2 - 1$ we then have then that the price function cannot be continuous at x_0^{**} and we then have x_0^{**} obtained by $x_0^{**} - \overline{x} = V_f(x_0^{**}, P^*(x_0^{**}))$ and $P^*(x_0^{**}) \in \arg\max_P V_f(x_0^{**}, P)$, and that the optimal price falls at the discontinuity, $\lim_{x_0 \nearrow x_0{**}} P^*(x_0) > x_0^{**} - \overline{x}$.

[22] This yields $a \approx 1.94$ and $2a^2 - 1 \approx 6.51$.

For the case in which the price function is continuous at x_0^{**} we can also obtain that the price function is not monotonic in x_0 for $x_0 \in [x_0^*, x_0^{**}]$. Note that $\frac{\partial^2 V_f(x_0)}{\partial P \partial x_0}_{|x_0=x_0^*} > 0$ so that the optimal price is increasing in x_0 for x_0 close to x_0^*. Note also that $\frac{\partial^2 V_f(x_0)}{\partial P \partial x_0}_{|x_0=x_0^{**}}$ when the price function is continuous at x_0^{**} can be negative if $\eta \overline{x} < \sqrt{1+\lambda/r}$. Using (8), we can then obtain that this condition always holds when the price function is continuous. In this case of $\beta \to 0$ and continuous price function, we then obtain that the optimal price is decreasing in x_0 for x_0 close to x_0^{**}.

Since $P^* < x_0 - \widetilde{x}$ for $x_0 > x_0^*$, the DM adopts the alternative at x_0 in the no-search mode. Since $P = x_0 - \overline{x}$ is optimal for $x_0 > x_0^{**}$, the DM adopts the alternative at x_0 in the search mode in that case. We can show that the firm's value function decreases in the price P when x_0 is high enough. When the DM's prior belief about the alternative is high enough, the firm can already obtain a high payoff by inducing the DM to adopt the alternative immediately without searching.

The Case of $\beta \to \infty$ for $x_0 \in [x_0^*, x_0^{**}]$:

When $\beta \to \infty$, we have $\overline{x}, \widetilde{x} \to \sqrt{\frac{\sigma^2}{2r}}$. Therefore, the interval $[x_0 - \overline{x}, x_0 - \widetilde{x}]$ disappears and the possible optimal prices are $P \geq x_0 - \widetilde{x}$. From the previous analysis, one can see that the optimal price is

$$P^* = \begin{cases} 1/\eta, \ if \ 1/\eta > x_0 - \widetilde{x} \\ x_0 - \widetilde{x}, \ otherwise. \end{cases}$$

To get that the price function is continuous for β large, we can obtain that $\frac{\partial^2 V_f(x,P)}{\partial P^2}_{|x=x_0^{**}, P=x_0^{**}-\overline{x}}$ is strictly negative for $\beta \to \infty$ and x_0^{**} satisfying $h(x_0^{**} - \overline{x}, x_0^{**}) = 0$. The result that the price function is monotonic for $\beta \to \infty$ is straightforward to obtain since we have $x_0^{**} - x_0^* \to 0$ for $\beta \to \infty$.

Optimal Pricing in a Two-period Model:

Consider a similar setup in discrete time. There are two periods, $t = 1, 2$. The DM can search for information about the alternative at most twice. Given the belief at the beginning of each period, x_t, the DM's belief will become $x_t + \Delta$ or $x_t - \Delta$ with equal probability if she is in the search mode and decides to search. The DM can adopt the alternative without searching, after searching once, or after searching twice. She may switch from the search to the no-search mode with probability λ at the end of the first period. The discount factor per period of both the firm and the DM is $\widehat{\delta}$. Let us first consider the optimal search strategy of the DM, where $y_t = x_t - P$.

Proposition 9. *Suppose $4\widehat{\delta} + (1-\lambda)\widehat{\delta}^2 > 4$.*[23] *If $y_0 \geq \frac{2(1-\lambda)\widehat{\delta}}{4-2\widehat{\delta}-2\lambda\widehat{\delta}-(1-\lambda)\widehat{\delta}^2}\Delta$ the DM adopts the*

[23] If this condition is not satisfied, the threshold of adopting the alternative without searching is different. But the intuition of the entire analysis is the same. We omit the presentation of that case for simplicity.

alternative without searching. If $y_0 \in [\Delta, \frac{2(1-\lambda)\widehat{\delta}}{4-2\widehat{\delta}-2\lambda\widehat{\delta}-(1-\lambda)\widehat{\delta}^2}\Delta)$ the DM adopts the alternative after receiving a positive signal, receiving a negative signal and then a positive signal, or switching to the no-search mode at the end of period one. If $y_0 \in [-\frac{1-\widehat{\delta}}{2-\widehat{\delta}}\Delta, \Delta)$ The DM adopts the alternative after receiving a positive signal or receiving a negative signal and then a positive signal. If $y_0 \in [-\Delta, -\frac{1-\widehat{\delta}}{2-\widehat{\delta}}\Delta)$ the DM adopts the alternative after receiving two positive signals, or receiving a positive signal and then switching to the no-search mode. If $y_0 \in [-2\Delta, -\Delta]$ the DM adopts the alternative after receiving two positive signals.

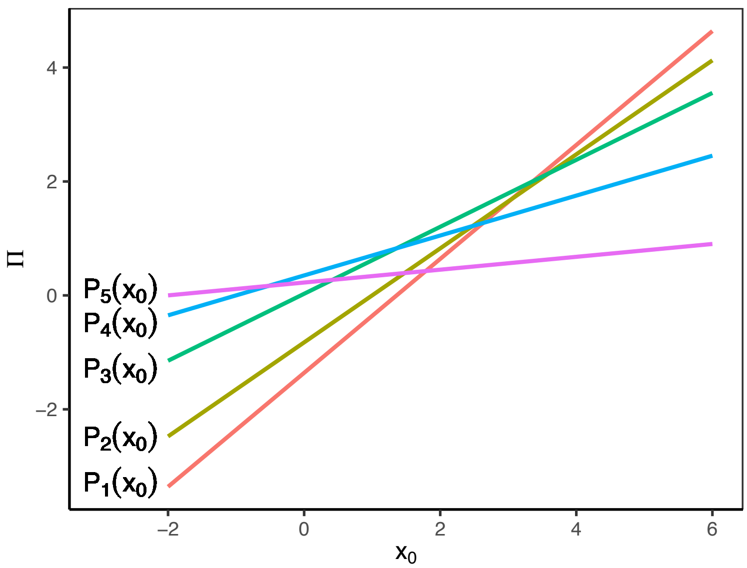

Figure A.1: Example of the firm's profit Π as a function of x_0 for $r = .95, \lambda = .5$, and $\Delta = 1$.

From this proposition, one can see that the DM's adoption likelihood is piecewise constant and non-decreasing in x_0. Therefore, the firm will only choose from prices such that $y_0 = x_0 - P \in \{\frac{2(1-\lambda)\widehat{\delta}}{4-2\widehat{\delta}-2\lambda\widehat{\delta}-(1-\lambda)\widehat{\delta}^2}\Delta, \Delta, -\frac{1-\widehat{\delta}}{2-\widehat{\delta}}\Delta, -\Delta, -2\Delta\}$. Denote those price schemes by $P_1(x_0) = x_0 - \frac{2(1-\lambda)\widehat{\delta}}{4-2\widehat{\delta}-2\lambda\widehat{\delta}-(1-\lambda)\widehat{\delta}^2}\Delta, P_2(x_0) = x_0 - \Delta, P_3(x_0) = x_0 + \frac{1-\widehat{\delta}}{2-\widehat{\delta}}\Delta, P_4(x_0) = x_0 + \Delta, P_5(x_0) = x_0 + 2\Delta$. Note that the price increases from $P_1(x_0)$ to $P_5(x_0)$ for a given x_0. The corresponding profits are: $\Pi_1(x_0) = x_0 - \frac{2(1-\lambda)\widehat{\delta}}{4-2\widehat{\delta}-2\lambda\widehat{\delta}-(1-\lambda)\widehat{\delta}^2}\Delta, \Pi_2(x_0) = (\frac{\widehat{\delta}}{2} + \frac{\lambda\widehat{\delta}}{2} + \frac{1-\lambda}{4}\widehat{\delta}^2)(x_0 - \Delta), \Pi_3(x_0) = (\frac{\widehat{\delta}}{2} + \frac{1-\lambda}{4}\widehat{\delta}^2)(x_0 + \frac{1-\widehat{\delta}}{2-\widehat{\delta}}\Delta), \Pi_4(x_0) = (\frac{\lambda\widehat{\delta}}{2} + \frac{1-\lambda}{4}\widehat{\delta}^2)(x_0 + \Delta), \Pi_5(x_0) = \frac{1-\lambda}{4}\widehat{\delta}^2(x_0 + 2\Delta)$. By plotting the firm's profits from all the candidate price schemes in Figure A.1, we can illustrate the optimal pricing strategy. The firm's expected payoff from charging each pricing scheme is linear in x_0. A lower pricing scheme leads to a higher adoption likelihood, and thus corresponds to a profit function with a higher slope and lower intercept. When the prior belief x_0 is low, the firm charges the highest candidate price $P_5(x_0)$, which increases in x_0 linearly. The intuition is that the DM only cares about $y_0 = x_0 - P$.

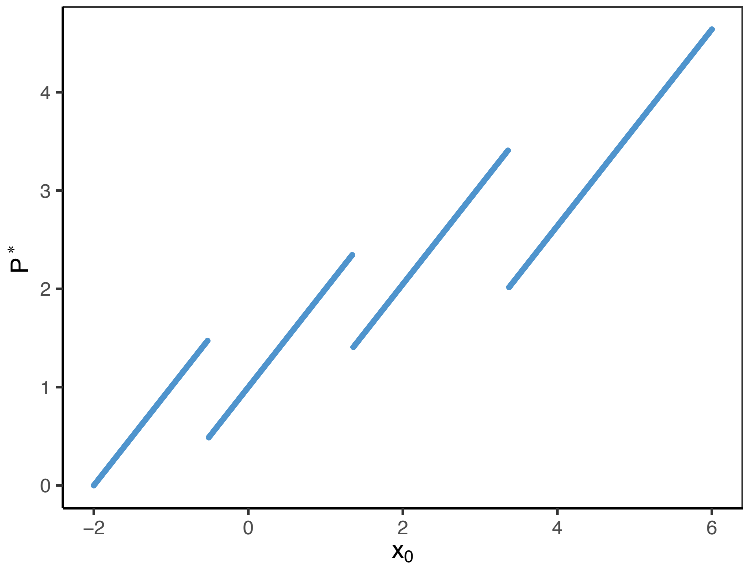

Figure A.2: Example of the optimal price P^* as a function of x_0 for $r = .95, \lambda = .5$, and $\Delta = 1$.

So, the firm can charge a higher price to induce the same adoption likelihood when the prior belief increases. As x_0 increases to a certain level, however, the firm switches from charging a price given by $P_5(x_0)$ to charging a lower price given by $P_4(x_0)$. Intuitively, as x_0 and the price increase, the firm's loss from non-adoption is larger. Therefore, the firm has a higher incentive to induce the DM to search less and adopt more while the cost of doing so, $P_5(x_0) - P_4(x_0)$, does not depend on x_0. When this incentive becomes strong enough, the optimal price has a discrete downward jump, as illustrated in Figure A.2. Then, the optimal price remains as given by $P_4(x_0)$ and increases in x_0 linearly until it switches from the pricing function $P_4(x_0)$ to $P_3(x_0)$ and decreases discontinuously. The optimal price then remains given $P_3(x_0)$ and increases in x_0 linearly until it switches from $P_3(x_0)$ to $P_1(x_0)$. For x_0 high enough, the optimal price is always given by $P_1(x_0)$, low enough such that the DM adopts the alternative without searching. In sum, each time the firm switches from one pricing scheme to another with a higher slope, the optimal price decreases discontinuously. In all other places, the optimal price increases in x_0 linearly.

The optimal price as a function of x_0 is smoother in continuous time. But non-monotonicity and discontinuity may still arise due to the effects we identify in discrete time.

Proof of Proposition 5:

When the initial belief is high, $x_0 > x_0^{**}$, Proposition 4 shows that the optimal price is $x_0 - \overline{x}$, and the DM adopts the alternative without searching. In this case, the firm's profit is $x_0 - \overline{x}$.

Proposition 1 shows that $\overline{x}$ increases in β. So, the firm's profit decreases in β. The firm does not retarget given any retargeting cost $k_r \geq 0$.

When the initial belief is low, $x_0 \leq 0$, Proposition 4 shows that the optimal price is $1/\eta$ and the DM does not adopt the alternative at x_0 in either the search mode or the no-search mode. The firm's profit is its value function evaluated at x_0. According to (17),

$$V_f(x_0) = \frac{2r + \lambda(e^{\widetilde{\eta}\delta} + e^{-\widetilde{\eta}\delta})}{(\widetilde{\eta} + \eta)e^{\eta(\widetilde{x}+P)+\widetilde{\eta}\delta} + (\widetilde{\eta} - \eta)e^{\eta(\widetilde{x}+P)-\widetilde{\eta}\delta}} \frac{\widetilde{\eta}P}{r + \lambda} e^{\eta x_0}.$$

One can see that $\eta = \sqrt{\frac{2r}{\sigma^2}\frac{r+\beta+\lambda}{r+\beta}}$ decreases in β. Because $x_0 \leq 0$, the last term of the value function $e^{\eta x_0}$ increases in β.

Now look at the terms before $e^{\eta x_0}$, $T_1 := \frac{2r+\lambda(e^{\widetilde{\eta}\delta}+e^{-\widetilde{\eta}\delta})}{(\widetilde{\eta}+\eta)e^{\eta(\widetilde{x}+P)+\widetilde{\eta}\delta}+(\widetilde{\eta}-\eta)e^{\eta(\widetilde{x}+P)-\widetilde{\eta}\delta}}\frac{\widetilde{\eta}P}{r+\lambda}$. One can see that $\lim_{\lambda\to+\infty} \eta/\widetilde{\eta} = \sqrt{r/(r+\beta)}$. One can also derive from (x) that $\lim_{\lambda\to+\infty} \widetilde{x}\sqrt{\lambda} = \beta\sigma/[\sqrt{2}(r + \sqrt{r(r+\beta)})]$. Therefore,

$$\begin{aligned}
\lim_{\lambda\to+\infty} \sqrt{\lambda}T_1 &= \lim_{\lambda\to+\infty} \sqrt{\lambda}\frac{2r + e^{\widetilde{\eta}\widetilde{x}}}{(\widetilde{\eta} + \eta)e^{1+\widetilde{\eta}\widetilde{x}}} \frac{\sqrt{(r+\beta)/r}}{r + \lambda} \\
&= \frac{\sqrt{(r+\beta)/r}}{(\sqrt{\frac{2}{\sigma^2}} + \sqrt{\frac{2r}{\sigma^2(r+\beta)}}) \cdot e} \\
&= \frac{\sigma}{\sqrt{2r}e} \frac{\sqrt{r+\beta}}{1 + \sqrt{\frac{r}{r+\beta}}},
\end{aligned}$$

which increases in β. Therefore, $V_f(x_0) = T_1 \cdot e^{\eta x_0}$ increases in β when λ is sufficiently large. Denote $\bar{k} = \frac{V_f(x_0|\beta_r) - V_f(x_0|\beta_0)}{\beta_r - \beta_0}$. One can see that $\bar{k} > 0$ and the firm retargets if and only if $k_r < \bar{k}$.

PROOF OF PROPOSITION 6:

When the initial belief is high, $x_0 > x_0^{**}$, Proposition 4 shows that the optimal price is $x_0 - \overline{x}$, and the DM adopts the alternative without searching. In this case, the firm's profit is $x_0 - \overline{x}$. Proposition 1 shows that $\overline{x}$ decreases in λ. So, the firm's profit increases in λ. The firm chooses $\lambda^* = \bar{\lambda}$.

When the initial belief is low, $x_0 \leq 0$, Proposition 4 shows that the optimal price is $1/\eta$ and the DM does not adopt the alternative at x_0 in either the search mode or the no-search mode. The firm's profit is its value function evaluated at x_0. According to (17),

$$V_f(x_0) = \frac{2r + \lambda(e^{\widetilde{\eta}\delta} + e^{-\widetilde{\eta}\delta})}{(\widetilde{\eta} + \eta)e^{\eta(\widetilde{x}+P)+\widetilde{\eta}\delta} + (\widetilde{\eta} - \eta)e^{\eta(\widetilde{x}+P)-\widetilde{\eta}\delta}} \frac{\widetilde{\eta}P}{r + \lambda} e^{\eta x_0}$$

One can see that $\eta = \sqrt{\frac{2r}{\sigma^2}\frac{r+\beta+\lambda}{r+\beta}}$ increases in λ. Because $x_0 \leq 0$, the last term of the value function

$e^{\eta x_0}$ decreases in λ.

Now look at the terms before $e^{\eta x_0}$, $T_1 = \frac{2r+\lambda(e^{\tilde\eta\delta}+e^{-\tilde\eta\delta})}{(\tilde\eta+\eta)e^{\eta(\tilde x+P)+\tilde\eta\delta}+(\tilde\eta-\eta)e^{\eta(\tilde x+P)-\tilde\eta\delta}}\frac{\tilde\eta P}{r+\lambda}$. We have shown in the main model that as $\beta\to 0, \tilde x\to 0, \eta\to\tilde\eta$ and $e^{\eta\overline{x}}(1-\eta\overline{x})+\lambda/r=0$. Therefore,

$$\begin{aligned}\lim_{\beta\to 0} T_1 &= \lim_{\beta\to 0}\frac{2r+\lambda(e^{\tilde\eta\overline{x}}+e^{-\tilde\eta\overline{x}})}{2\tilde\eta e^{1+\tilde\eta\overline{x}}(r+\lambda)}\\ &= \lim_{\beta\to 0}\frac{2r+\lambda[\frac{\lambda}{r(\tilde\eta\overline{x}-1)}+\frac{r(\tilde\eta\overline{x}-1)}{\lambda}]}{2\tilde\eta(r+\lambda)\frac{\lambda e}{r(\tilde\eta\overline{x}-1)}}\\ &= K\frac{(\lambda-r)\sigma^2+2r^2\overline{x}^2}{\lambda\sqrt{\lambda+r}},\end{aligned}$$

for some $K>0$. Denote the last expression by $B(\lambda)$, and denote $K\frac{(\lambda-r)\sigma^2+2r^2k^2}{\lambda\sqrt{\lambda+r}}$ by $\widetilde{B}(\lambda)$, where $k>0$ is an arbitrary fixed constant.

$$\widetilde{B}'(\lambda) = K\frac{(-\lambda^2+3\lambda r+2r^2)\sigma^2-2r^2(3\lambda+2r)k^2}{2\lambda^2(\lambda+r)^{\frac{3}{2}}}$$

One can see that $-\lambda^2+3\lambda r+2r^2<0$, and thus $\widetilde{B}'(\lambda)<0$, when $r\le(\sqrt{17}-3)\lambda/4$. Therefore, $\widetilde{B}(\lambda)$ decreases in λ if $r\le(\sqrt{17}-3)\lambda/4$. Because $\overline{x}$ decreases in λ, when we replace the constant k in $\widetilde{B}(\lambda)$ by $\overline{x}$ in $B(\lambda)$, we also have $B(\lambda)$ decreases in λ if $r\le(\sqrt{17}-3)\lambda/4$. Therefore, $V_f(x_0)=T_1\cdot e^{\eta x_0}$ decreases in λ when β is sufficiently small and r is small. The firm chooses $\lambda^*=\underline{\lambda}$.

Derivation of the Optimal Decision-Making in the Two Search Modes Case:

Applying Itô's Lemma to (19) and solve the differential equation, we can obtain

$$V_1(x) = B_3e^{\tilde\eta x}+B_4e^{-\tilde\eta x}+\frac{\lambda}{r+\lambda}x, \tag{xxii}$$

where B_3 and B_4 are constants to be determined.

Applying Itô's Lemma to (21) and solve the differential equation, we can obtain

$$V_2(x) = B_1e^{\tilde\eta x}+B_2e^{-\tilde\eta x}+\frac{\lambda}{r+\lambda}x, \tag{xxiii}$$

where B_1 and B_2 are constants to be determined. We can then use (xxiii) in (20) to obtain that for $x\in(\widetilde{x},\underline{x})$, solving the corresponding differential equation,

$$V_1(x) = B_5e^{\tilde\eta x}+B_6e^{-\tilde\eta x}+\frac{\lambda^2}{(r+\lambda)^2}x+\frac{\lambda\tilde\eta}{2(r+\lambda)}x[B_2e^{-\tilde\eta x}-B_1e^{\tilde\eta x}], \tag{xxiv}$$

where B_5 and B_6 are constant to be determined.

Putting together (20) and (22) for $x < \widetilde{x}$, we obtain a system of differential equations

$$(r+\lambda)V_2(x) = \frac{\sigma^2}{2}V_2''(x) + \lambda\frac{\beta}{r+\beta}V_1(x) \tag{xxv}$$

$$(r+\lambda)V_1(x) = \frac{\sigma^2}{2}V_1''(x) + \lambda V_2(x) \tag{xxvi}$$

which has the solution

$$V_2(x) = \widetilde{B}_1 e^{z_1 x} + \widetilde{B}_2 e^{z_2 x} \tag{xxvii}$$

$$V_1(x) = \sqrt{\frac{r+\beta}{\beta}}\left[\widetilde{B}_2 e^{z_2 x} - \widetilde{B}_1 e^{z_1 x}\right] \tag{xxviii}$$

where $z_1 = \sqrt{\widetilde{\eta}^2 + \frac{2\lambda}{\sigma^2}\sqrt{\frac{\beta}{r+\beta}}}$, and $z_2 = \sqrt{\widetilde{\eta}^2 - \frac{2\lambda}{\sigma^2}\sqrt{\frac{\beta}{r+\beta}}}$, and $\widetilde{B}_1$ and $\widetilde{B}_2$ are constants to be determined, where we use that $\lim_{x\to-\infty} V_1(x) = \lim_{x\to-\infty} V_2(x) = 0$.

Value matching and smooth pasting at the different thresholds, $V_1(\overline{x}) = \overline{x}, V_1'(\overline{x}) = 1, V_1(\underline{x}^+) = V_1(\underline{x}^-), V_1'(\underline{x}^+) = V_1'(\underline{x}^-), V_1(\widetilde{x}^+) = V_1(\widetilde{x}^-), V_1'(\widetilde{x}^+) = V_1'(\widetilde{x}^-), V_2(\widetilde{x}^+) = V_2(\widetilde{x}^-), V_2'(\widetilde{x}^+) = V_2'(\widetilde{x}^-), V_2(\underline{x}) = \underline{x}, V_2'(\underline{x}) = 1, \frac{\beta}{r+\beta}V_1(\widetilde{x}) = \widetilde{x}$, lead to the following system of 11 equations to obtain the 11 unknowns, $\overline{x}_1, \overline{x}_2, \widetilde{x}, B_1, B_2, B_3, B_4, B_5, B_6, \widetilde{B}_1$, and $\widetilde{B}_2$.

$$\begin{aligned} B_3\underline{X} + B_4/\underline{X} + \frac{\lambda}{r+\lambda}\underline{x} &= B_5\underline{X} + B_6/\underline{X} + \frac{\lambda^2}{(r+\lambda)^2}\underline{x} - \frac{\lambda\widetilde{\eta}}{2(r+\lambda)}B_1\underline{x}\underline{X} \\ &\quad + \frac{\lambda\widetilde{\eta}}{2(r+\lambda)}B_2\underline{x}/\underline{X} \end{aligned} \tag{xxix}$$

$$\begin{aligned} \widetilde{\eta}B_3\underline{X} - \widetilde{\eta}B_4/\underline{X} + \frac{\lambda}{r+\lambda} &= \widetilde{\eta}B_5\underline{X} - \widetilde{\eta}B_6/\underline{X} + \frac{\lambda^2}{(r+\lambda)^2} - \frac{\lambda\widetilde{\eta}}{2(r+\lambda)}B_1\underline{X} - \frac{\lambda\widetilde{\eta}^2}{2(r+\lambda)}B_1\underline{x}\underline{X} \\ &\quad + \frac{\lambda\widetilde{\eta}}{2(r+\lambda)}B_2/\underline{X} - \frac{\lambda\widetilde{\eta}^2}{2(r+\lambda)}B_2\underline{x}/\underline{X} \end{aligned} \tag{xxx}$$

$$\begin{aligned} B_5\widetilde{X} + B_6/\widetilde{X} + \frac{\lambda^2}{(r+\lambda)^2}\widetilde{x} &- \frac{\lambda\widetilde{\eta}}{2(r+\lambda)}B_1\widetilde{x}\widetilde{X} + \frac{\lambda\widetilde{\eta}}{2(r+\lambda)}B_2\widetilde{x}/\widetilde{X} = \\ &\quad \sqrt{\frac{r+\beta}{\beta}}\left[\widetilde{B}_2 e^{z_2\widetilde{x}} - \widetilde{B}_1 e^{z_1\widetilde{x}}\right] \end{aligned} \tag{xxxi}$$

$$\begin{aligned} \widetilde{\eta}B_5\widetilde{X} - \widetilde{\eta}B_6/\widetilde{X} + \frac{\lambda^2}{(r+\lambda)^2} &- \frac{\lambda\widetilde{\eta}}{2(r+\lambda)}B_1\widetilde{X} - \frac{\lambda\widetilde{\eta}^2}{2(r+\lambda)}B_1\widetilde{x}\widetilde{X} + \frac{\lambda\widetilde{\eta}}{2(r+\lambda)}B_2/\widetilde{X} \\ &- \frac{\lambda\widetilde{\eta}^2}{2(r+\lambda)}B_2\widetilde{x}/\widetilde{X} = \sqrt{\frac{r+\beta}{\beta}}\left[z_2\widetilde{B}_2 e^{z_2\widetilde{x}} - z_1\widetilde{B}_1 e^{z_1\widetilde{x}}\right] \end{aligned} \tag{xxxii}$$

$$
\begin{aligned}
B_1\widetilde{X} + B_2/\widetilde{X} + \frac{\lambda}{r+\lambda}\widetilde{x} &= \widetilde{B}_1 e^{z_1\widetilde{x}} + \widetilde{B}_2 e^{z_2\widetilde{x}} && \text{(xxxiii)}\\
\widetilde{\eta}B_1\widetilde{X} - \widetilde{\eta}B_2/\widetilde{X} + \frac{\lambda}{r+\lambda} &= z_1\widetilde{B}_1 e^{z_1\widetilde{x}} + z_2\widetilde{B}_2 e^{z_2\widetilde{x}} && \text{(xxxiv)}\\
B_3\overline{X} + B_4/\overline{X} + \frac{\lambda}{r+\lambda}\overline{x} &= \overline{x} && \text{(xxxv)}\\
\widetilde{\eta}B_3\overline{X} - \widetilde{\eta}B_4/\overline{X} + \frac{\lambda}{r+\lambda} &= 1 && \text{(xxxvi)}\\
B_1\underline{X} + B_2/\underline{X} + \frac{\lambda}{r+\lambda}\underline{x} &= \underline{x} && \text{(xxxvii)}\\
\widetilde{\eta}B_1\underline{X} - \widetilde{\eta}B_2/\underline{X} + \frac{\lambda}{r+\lambda} &= 1 && \text{(xxxviii)}\\
\sqrt{\frac{\beta}{r+\beta}}\left(-\widetilde{B}_1 e^{z_1\widetilde{x}} + \widetilde{B}_2 e^{z_2\widetilde{x}}\right) &= \widetilde{x}, && \text{(xxxix)}
\end{aligned}
$$

where $\overline{X} = e^{\widetilde{\eta}\overline{x}}$, $\underline{X} = e^{\widetilde{\eta}\underline{x}}$, and $\widetilde{X} = e^{\widetilde{\eta}\widetilde{x}}$.

Putting together (xxxv) and (xxxvi) one obtains

$$
\begin{aligned}
2B_3\overline{X} &= \frac{r}{r+\lambda}\left(\overline{x} + \frac{1}{\widetilde{\eta}}\right) && \text{(xl)}\\
2B_4/\overline{X} &= \frac{r}{r+\lambda}\left(\overline{x} - \frac{1}{\widetilde{\eta}}\right). && \text{(xli)}
\end{aligned}
$$

Putting together (xxxvii) and (xxxviii) one obtains

$$
\begin{aligned}
2B_1\underline{X} &= \frac{r}{r+\lambda}\left(\underline{x} + \frac{1}{\widetilde{\eta}}\right) && \text{(xlii)}\\
2B_2/\underline{X} &= \frac{r}{r+\lambda}\left(\underline{x} - \frac{1}{\widetilde{\eta}}\right). && \text{(xliii)}
\end{aligned}
$$

Putting together (xxix) and (xxx) one obtains

$$
\begin{aligned}
2B_3\underline{X} + \frac{\lambda}{r+\lambda}\left(\underline{x} + \frac{1}{\widetilde{\eta}}\right) = 2B_5\underline{X} &+ \frac{\lambda^2}{(r+\lambda)^2}\left(\underline{x} + \frac{1}{\widetilde{\eta}}\right) - \frac{\lambda}{2(r+\lambda)}B_1\underline{X}(1+2\widetilde{\eta}\underline{x})\\
&+ \frac{\lambda}{2(r+\lambda)}B_2/\underline{X} && \text{(xliv)}\\
2B_4/\underline{X} + \frac{\lambda}{r+\lambda}\left(\underline{x} - \frac{1}{\widetilde{\eta}}\right) = 2B_6/\underline{X} &+ \frac{\lambda^2}{(r+\lambda)^2}\left(\underline{x} - \frac{1}{\widetilde{\eta}}\right) - \frac{\lambda}{2(r+\lambda)}B_2/\underline{X}(1-2\widetilde{\eta}\underline{x})\\
&+ \frac{\lambda}{2(r+\lambda)}B_1\underline{X}. && \text{(xlv)}
\end{aligned}
$$

Putting together (xxxi) and (xxxii) one obtains

$$
\begin{aligned}
2B_5\widetilde{X} + \frac{\lambda^2}{(r+\lambda)^2}\left(\widetilde{x}+\frac{1}{\widetilde{\eta}}\right) \quad - \quad & \frac{\lambda}{2(r+\lambda)}B_1\widetilde{X}(1+2\widetilde{\eta}\widetilde{x}) + \frac{\lambda}{2(r+\lambda)}B_2/\widetilde{X} = \\
& \sqrt{\frac{r+\beta}{\beta}}\left(-\widetilde{\widetilde{B}}_1\left(1+\frac{z_1}{\widetilde{\eta}}\right)+\widetilde{\widetilde{B}}_2\left(1+\frac{z_2}{\widetilde{\eta}}\right)\right) \qquad \text{(xlvi)} \\
2B_6/\widetilde{X} + \frac{\lambda^2}{(r+\lambda)^2}\left(\widetilde{x}-\frac{1}{\widetilde{\eta}}\right) \quad - \quad & \frac{\lambda}{2(r+\lambda)}B_2/\widetilde{X}(1-2\widetilde{\eta}\widetilde{x}) + \frac{\lambda}{2(r+\lambda)}B_1\widetilde{X} = \\
& \sqrt{\frac{r+\beta}{\beta}}\left(-\widetilde{\widetilde{B}}_1\left(1-\frac{z_1}{\widetilde{\eta}}\right)+\widetilde{\widetilde{B}}_2\left(1-\frac{z_2}{\widetilde{\eta}}\right)\right) \qquad \text{(xlvii)}
\end{aligned}
$$

where $\widetilde{\widetilde{B}}_1 = \widetilde{B}_1 e^{z_1\widetilde{x}}$ and $\widetilde{\widetilde{B}}_2 = \widetilde{B}_2 e^{z_2\widetilde{x}}$. Putting together (xxxiii) and (xxxiv) one obtains

$$
\begin{aligned}
2B_1\widetilde{X} + \frac{\lambda}{r+\lambda}\left(\widetilde{x}+\frac{1}{\widetilde{\eta}}\right) &= \widetilde{\widetilde{B}}_1\left(1+\frac{z_1}{\widetilde{\eta}}\right)+\widetilde{\widetilde{B}}_2\left(1+\frac{z_2}{\widetilde{\eta}}\right) \qquad \text{(xlviii)} \\
2B_2/\widetilde{X} + \frac{\lambda}{r+\lambda}\left(\widetilde{x}-\frac{1}{\widetilde{\eta}}\right) &= \widetilde{\widetilde{B}}_1\left(1-\frac{z_1}{\widetilde{\eta}}\right)+\widetilde{\widetilde{B}}_2\left(1-\frac{z_2}{\widetilde{\eta}}\right). \qquad \text{(xlix)}
\end{aligned}
$$

Using (xxxix) we obtain $\widetilde{\widetilde{B}}_2 = \widetilde{\widetilde{B}}_1 + \sqrt{\frac{r+\beta}{\beta}}\widetilde{x}$, which we can then substitute in (xlvi)-(xlix). Using the resulting equations (xlviii) and (xlix) we can obtain

$$
\begin{aligned}
\frac{2\widetilde{\eta}-z_1-z_2}{2\widetilde{\eta}+z_1+z_2}\left[2B_1\widetilde{X} + \frac{\lambda}{r+\lambda}\left(\widetilde{x}+\frac{1}{\widetilde{\eta}}\right) - \sqrt{\frac{r+\beta}{\beta}}\widetilde{x}\left(1+\frac{z_2}{\widetilde{\eta}}\right)\right] &= \\
2B_2/\widetilde{X} + \frac{\lambda}{r+\lambda}\left(\widetilde{x}-\frac{1}{\widetilde{\eta}}\right) - \sqrt{\frac{r+\beta}{\beta}}\widetilde{x}\left(1-\frac{z_2}{\widetilde{\eta}}\right). & \qquad \text{(l)}
\end{aligned}
$$

Using (xlii) and (xliii) in (l) one can then obtain

$$
\begin{aligned}
\frac{2\widetilde{\eta}-z_1-z_2}{2\widetilde{\eta}+z_1+z_2}\left[\frac{\widetilde{X}}{\underline{X}}\frac{r}{r+\lambda}\left(\underline{x}+\frac{1}{\widetilde{\eta}}\right) + \frac{\lambda}{r+\lambda}\left(\widetilde{x}+\frac{1}{\widetilde{\eta}}\right) - \sqrt{\frac{r+\beta}{\beta}}\widetilde{x}\left(1+\frac{z_2}{\widetilde{\eta}}\right)\right] &= \\
\frac{\underline{X}}{\widetilde{X}}\frac{r}{r+\lambda}\left(\underline{x}-\frac{1}{\widetilde{\eta}}\right) + \frac{\lambda}{r+\lambda}\left(\widetilde{x}-\frac{1}{\widetilde{\eta}}\right) - \sqrt{\frac{r+\beta}{\beta}}\widetilde{x}\left(1-\frac{z_2}{\widetilde{\eta}}\right), & \qquad \text{(li)}
\end{aligned}
$$

which is an equation on only $\underline{x}$ and $\widetilde{x}$. Note that when $\beta \to \infty$ we have $\underline{x}, \widetilde{x} \to \sqrt{\frac{\sigma^2}{2r}}$ and (li) is satisfied.

Let $\delta_1 = \underline{x} - \widetilde{x}, \delta_2 = \overline{x} - \underline{x}, D_1 = e^{\widetilde{\eta}\delta_1}$, and $D_2 = e^{\widetilde{\eta}\delta_2}$. Using (xliv) and (xlvi) to take out B_5, and using B_1 from (xlii), B_2 from (xliii), B_3 from (xl), and $\widetilde{C}_1$ from (xlviii), we can obtain

$$
\frac{1}{D_2}\left(\overline{x}+\frac{1}{\widetilde{\eta}}\right) = \frac{\lambda+r}{r}G_1(\underline{x},\widetilde{x}), \qquad \text{(lii)}
$$

where

$$G_1(\underline{x},\widetilde{x}) \quad = \quad D_1\Big[-\frac{\lambda^2}{(r+\lambda)^2}\Big(\widetilde{x}+\frac{1}{\widetilde{\eta}}\Big)+\frac{r+\beta}{\beta}\widetilde{x}\Big(1+\frac{z_2}{\widetilde{\eta}}\Big)+\sqrt{\frac{r+\beta}{\beta}}\frac{z_2-z_1}{2\widetilde{\eta}+z_1+z_2}\Big[\frac{1}{D_1}\frac{r}{r+\lambda}\Big(\underline{x}+\frac{1}{\widetilde{\eta}}\Big)+ \frac{\lambda}{r+\lambda}\Big(\widetilde{x}+\frac{1}{\widetilde{\eta}}\Big)-\widetilde{x}\sqrt{\frac{r+\beta}{\beta}}\Big(1+\frac{z_2}{\widetilde{\eta}}\Big)\Big]\Big]-\frac{\lambda r}{4(r+\lambda)^2}\Big(\underline{x}(3+2\widetilde{\eta}\delta_1+D_1^2)+ \frac{1}{\widetilde{\eta}}(5+2\widetilde{\eta}\delta_1-D_1^2)\Big). \tag{liii}$$

Similarly, using (xlv) and (xlvii) to take out B_6, and using B_1 from (xlii), B_2 from (xliii), B_4 from (xli), and $\widetilde{C}_1$ from (xlviii), we can obtain

$$D_2\Big(\overline{x}-\frac{1}{\widetilde{\eta}}\Big)=\frac{\lambda+r}{r}G_2(\underline{x},\widetilde{x}), \tag{liv}$$

where

$$G_2(\underline{x},\widetilde{x}) \quad = \quad \frac{1}{D_1}\Big[-\frac{\lambda^2}{(r+\lambda)^2}\Big(\widetilde{x}-\frac{1}{\widetilde{\eta}}\Big)+\frac{r+\beta}{\beta}\widetilde{x}\Big(1-\frac{z_2}{\widetilde{\eta}}\Big)+\sqrt{\frac{r+\beta}{\beta}}\frac{z_1-z_2}{2\widetilde{\eta}+z_1+z_2}\Big[\frac{1}{D_1}\frac{r}{r+\lambda}\Big(\underline{x}+ \frac{1}{\widetilde{\eta}}\Big)+\frac{\lambda}{r+\lambda}\Big(\widetilde{x}+\frac{1}{\widetilde{\eta}}\Big)-\widetilde{x}\sqrt{\frac{r+\beta}{\beta}}\Big(1+\frac{z_2}{\widetilde{\eta}}\Big)\Big]\Big]-\frac{\lambda r}{4(r+\lambda)^2}\Big(\underline{x}(3-2\widetilde{\eta}\delta_1+\frac{1}{D_1^2})-\frac{1}{\widetilde{\eta}}(5- 2\widetilde{\eta}\delta_1-\frac{1}{D_1^2})\Big). \tag{lv}$$

Note then that (li), (lii), and (liv) is a system of equations for $\overline{x}, \underline{x}$, and $\widetilde{x}$. Note also that putting (lii) and (liv) together one obtains

$$\overline{x}^2=\frac{(\lambda+r)^2}{r^2}G_1G_2+\frac{1}{\widetilde{\eta}^2}, \tag{lvi}$$

which determines $\overline{x}$ as a function of $\underline{x}$ and $\widetilde{x}$. Plugging it in (lii), we can then use (li) and (lii) to solve for $\underline{x}$ and $\widetilde{x}$.

Derivation of Optimal Decision-Making for $\beta = 0$ in the Two Search Modes Case:

In the case of $\beta \to 0$ and $\widetilde{x} \to 0$, we obtain $z_1, z_2 \to \widetilde{\eta}$, and for $x < \widetilde{x} = 0$ we obtain

$$V_2(x) \quad = \quad \widehat{B}_1 e^{\widetilde{\eta}x} \tag{lvii}$$

$$V_1(x) \quad = \quad \widehat{B}_2 e^{\widetilde{\eta}x}-\frac{\lambda\widehat{B}_1}{\sigma^2\widetilde{\eta}}e^{\widetilde{\eta}x}, \tag{lviii}$$

where $\widehat{B}_1$ and $\widehat{B}_2$ are constants to be determined.

We then have that the condition $\frac{\beta}{r+\beta}V_1(\widetilde{x})=\widetilde{x}$, (xxxix), is no longer required, and that conditions (xxxi)-(xxxiv), are replaced by the conditions

$$\begin{aligned} B_5 + B_6 &= \widehat{B}_2 && \text{(lix)} \\ \widetilde{\eta}B_5 - \widetilde{\eta}B_6 + \frac{\lambda^2}{(r+\lambda)^2} &- \frac{\lambda\widetilde{\eta}}{2(r+\lambda)}B_1 + \frac{\lambda\widetilde{\eta}}{2(r+\lambda)}B_2 \\ &= \widetilde{\eta}\widehat{B}_2 - \frac{\lambda\widehat{B}_1}{\sigma^2\widetilde{\eta}} && \text{(lx)} \\ B_1 + B_2 &= \widehat{B}_1 && \text{(lxi)} \\ \widetilde{\eta}B_1 - \widetilde{\eta}B_2 + \frac{\lambda}{r+\lambda} &= \widetilde{\eta}\widehat{B}_1, && \text{(lxii)} \end{aligned}$$

respectively.

Using (lxi) and (lxii) we can obtain $B_2 = \frac{\lambda}{2\widetilde{\eta}(r+\lambda)}$. Using this in (xliii), we can obtain the condition for the optimal $\underline{x}$ as

$$e^{\eta\underline{x}}(1-\eta) + \frac{\lambda}{r} = 0, \tag{lxiii}$$

as $\eta = \widetilde{\eta}$ for $\beta = 0$, which is intuitively the same condition as (8). Using (xlii) and (xxxiii) we can then also obtain $\widehat{B}_1 = \frac{r}{2(r+\lambda)}\left(\underline{x} + \frac{1}{\widetilde{\eta}}\right)\frac{1}{\underline{X}} + \frac{\lambda}{2\widetilde{\eta}(r+\lambda)}$.

Note also that in this case (xlvii) is replaced by

$$2B_6 - \frac{\lambda^2}{\widetilde{\eta}(r+\lambda^2)} - \frac{\lambda}{2(r+\lambda)}B_2 + \frac{\lambda}{2(r+\lambda)}B_1 = \frac{\lambda\widehat{B}_1}{2(r+\lambda)}. \tag{lxiv}$$

Using (lxiv) and (xlv) to substitute away B_6, we can then use B_1, B_2, and B_4 obtained above to yield

$$\frac{\lambda}{4\widetilde{\eta}} + \frac{\lambda}{\widetilde{\eta}} + \frac{r}{4}\underline{X}\left(\underline{x} + \frac{1}{\widetilde{\eta}}\right) = \frac{r+\lambda}{\lambda}\overline{X}\left(\overline{x} - \frac{1}{\widetilde{\eta}}\right) + \frac{\lambda(1-\lambda)(r+\lambda)}{r\widetilde{\eta}} - \frac{\lambda}{2}\underline{x}, \tag{lxv}$$

which determines $\overline{x}$ as a function of $\underline{x}$.

Proof of Proposition 7:

We can obtain the condition for the optimal $\underline{x}$ as

$$e^{\eta\underline{x}}(1-\eta\underline{x}) + \frac{\lambda}{r} = 0, \tag{lxvi}$$

as $\eta = \widetilde{\eta}$ for $\beta = 0$, which is intuitively the same condition as (8). Furthermore, we can obtain

$$\frac{\lambda}{4\widetilde{\eta}} + \frac{\lambda}{\widetilde{\eta}} + \frac{r}{4}\underline{X}\left(\underline{x} + \frac{1}{\widetilde{\eta}}\right) = \frac{r+\lambda}{\lambda}\overline{X}\left(\overline{x} - \frac{1}{\widetilde{\eta}}\right) + \frac{\lambda(1-\lambda)(r+\lambda)}{r\widetilde{\eta}} - \frac{\lambda}{2}\underline{x}, \tag{lxvii}$$

where $\overline{X} = e^{\widetilde{\eta}\overline{x}}$ and $\underline{X} = e^{\widetilde{\eta}\underline{x}}$, which determines $\overline{x}$ as a function of $\underline{x}$. For $\lambda \to 0$, we can then obtain

$\underline{x}, \overline{x} \to \sqrt{\frac{\sigma^2}{2r}}$, and

$$\frac{\underline{x} - 1/\eta}{\lambda} \to \frac{1}{r\eta e} \tag{lxviii}$$

$$\frac{\overline{x} - 1/\eta}{\lambda} \to \frac{1}{2r\eta}. \tag{lxix}$$

For $\beta = 0$ and λ sufficiently small, the extent of choice closure in search mode 1 is approximated by

$$\overline{x} - \underline{x} \approx \frac{\lambda}{r\eta}\left(\frac{1}{2} - \frac{1}{e}\right) = \sqrt{\frac{\lambda}{r+\lambda}} \frac{\sigma^2}{2r} \frac{1}{\sqrt{r}} \left(\frac{1}{2} - \frac{1}{e}\right) \tag{lxx}$$

The extent of choice closure in search mode 2 is approximated by

$$\underline{x} - \widetilde{x} \approx \frac{1}{\eta}\left(\frac{\lambda}{re} + 1\right) = \frac{\sigma^2}{2r}\left(\sqrt{\frac{\lambda}{r+\lambda}} \frac{1}{\sqrt{r}} \frac{1}{e} + \sqrt{\frac{r+\lambda}{r}}\right) \tag{lxxi}$$

The comparative statics follow immediately.

Derivation of Solution for Start-Up Search Costs Case:

Using Itô's Lemma on equation (24) and solving the corresponding differential equation, we can obtain

$$\widetilde{V}(x) = \frac{\lambda F + c}{\sigma^2} x^2 + a_1 x + a_0, \tag{lxxii}$$

where a_1 and a_0 are constants to be determined.

Using Itô's Lemma on equation (25) and solving the corresponding differential equation, one obtains

$$V(x) = C_1 e^{\widehat{\eta} x} + C_2 e^{-\widehat{\eta} x} + x - c/\lambda, \tag{lxxiii}$$

Using Itô's Lemma on equation (26) and solving the corresponding differential equation, one obtains

$$\widehat{V}(x) = C_3 e^{\widehat{\eta} x} + C_4 e^{-\widehat{\eta} x} - c/\lambda. \tag{lxxiv}$$

If $\widetilde{x} > 0$, then value matching and smooth pasting at $\overline{x}, \widetilde{x}, \widehat{x}$, and $\underline{\widehat{x}}$ leads to $V(\overline{x}) = \overline{x}, V'(\overline{x}) = 1, V(\widetilde{x}) = \widetilde{V}(\widetilde{x}), V'(\widetilde{x}) = \widetilde{V}'(\widetilde{x}), V(\widetilde{x}) - F = \widetilde{x}, \widetilde{V}(\widehat{x}) - F = 0, \widetilde{V}(\widehat{x}) = \widehat{V}(\widehat{x}), \widetilde{V}'(\widehat{x}) = \widehat{V}'(\widehat{x}), \widehat{V}(\underline{\widehat{x}}) = 0,$

and $\widehat{V}'(\underline{\widehat{x}}) = 0$, which are the conditions

$$\begin{aligned} C_1 e^{\widehat{\eta}\overline{x}} + C_2 e^{-\widehat{\eta}\overline{x}} &= c/\lambda && \text{(lxxv)} \\ C_1 e^{\widehat{\eta}\overline{x}} - C_2 e^{-\widehat{\eta}\overline{x}} &= 0 && \text{(lxxvi)} \\ C_1 e^{\widehat{\eta}\widetilde{x}} + C_2 e^{-\widehat{\eta}\widetilde{x}} &= F + c/\lambda && \text{(lxxvii)} \\ \widehat{\eta}[C_1 e^{\widehat{\eta}\widetilde{x}} - C_2 e^{-\widehat{\eta}\widetilde{x}}] + 1 &= a_1 + 2\frac{\lambda F + c}{\sigma^2}\widetilde{x} && \text{(lxxviii)} \\ C_3 e^{\widehat{\eta}\widehat{x}} + C_4 e^{-\widehat{\eta}\widehat{x}} &= F + c/\lambda && \text{(lxxix)} \\ \widehat{\eta}[C_3 e^{\widehat{\eta}\widehat{x}} - C_4 e^{-\widehat{\eta}\widehat{x}}] &= a_1 + 2\frac{\lambda F + c}{\sigma^2}\widehat{x} && \text{(lxxx)} \\ C_3 e^{\widehat{\eta}\underline{\widehat{x}}} + C_4 e^{-\widehat{\eta}\underline{\widehat{x}}} &= c/\lambda && \text{(lxxxi)} \\ C_3 e^{\widehat{\eta}\underline{\widehat{x}}} - C_4 e^{-\widehat{\eta}\underline{\widehat{x}}} &= 0 && \text{(lxxxii)} \\ a_0 + a_1\widetilde{x} + \frac{\lambda F + c}{\sigma^2}\widetilde{x}^2 &= \widetilde{x} + F && \text{(lxxxiii)} \\ a_0 + a_1\widehat{x} + \frac{\lambda F + c}{\sigma^2}\widehat{x}^2 &= F. && \text{(lxxxiv)} \end{aligned}$$

From (lxxv) and (lxxvi) we can obtain $C_1 = \frac{c}{2\lambda}e^{-\widehat{\eta}\overline{x}}$ and $C_2 = \frac{c}{2\lambda}e^{\widehat{\eta}\overline{x}}$. Similarly, (lxxxi) and (lxxxii) we can obtain $C_3 = \frac{c}{2\lambda}e^{-\widehat{\eta}\underline{\widehat{x}}}$ and $C_4 = \frac{c}{2\lambda}e^{\widehat{\eta}\underline{\widehat{x}}}$. Using this in the other equations we can then obtain

$$\begin{aligned} \frac{c}{2\lambda}e^{\widehat{\eta}(\widetilde{x}-\overline{x})} + \frac{c}{2\lambda}e^{-\widehat{\eta}(\widetilde{x}-\overline{x})} &= F + \frac{c}{\lambda} && \text{(lxxxv)} \\ \widehat{\eta}\left[\frac{c}{2\lambda}e^{\widehat{\eta}(\widetilde{x}-\overline{x})} - \frac{c}{2\lambda}e^{-\widehat{\eta}(\widetilde{x}-\overline{x})}\right] + 1 &= a_1 + 2\frac{\lambda F + c}{\sigma^2}\widetilde{x} && \text{(lxxxvi)} \\ \frac{c}{2\lambda}e^{\widehat{\eta}(\widehat{x}-\underline{\widehat{x}})} + \frac{c}{2\lambda}e^{-\widehat{\eta}(\widehat{x}-\underline{\widehat{x}})} &= F + \frac{c}{\lambda} && \text{(lxxxvii)} \\ \widehat{\eta}\left[\frac{c}{2\lambda}e^{\widehat{\eta}(\widehat{x}-\underline{\widehat{x}})} - \frac{c}{2\lambda}e^{-\widehat{\eta}(\widehat{x}-\underline{\widehat{x}})}\right] &= a_1 + 2\frac{\lambda F + c}{\sigma^2}\widehat{x} && \text{(lxxxviii)} \\ \frac{\lambda F + c}{\sigma^2}(\widetilde{x}^2 - \widehat{x}^2) + a_1(\widetilde{x} - \widehat{x}) &= \widetilde{x}. && \text{(lxxxix)} \end{aligned}$$

From (lxxxv) and (lxxxvii) we can obtain $\overline{x} - \widetilde{x} = \widehat{x} - \underline{\widehat{x}}$. Using (lxxxv) we can also obtain $e^{\widehat{\eta}(\widetilde{x}-\overline{x})} = 1/H$, where

$$H = 1 + \frac{\lambda F}{c} + \sqrt{\left(\frac{\lambda F}{c} + 1\right)^2 - 1}. \tag{xc}$$

Using $\overline{x} - \widetilde{x} = \widehat{x} - \underline{\widehat{x}}$ in (lxxxvi) and (lxxxviii) we can obtain

$$a_1 = \frac{1}{2} - \frac{\lambda F + c}{\sigma^2}(\widetilde{x} + \widehat{x}). \tag{xci}$$

Substituting in (lxxxix) one obtains $\widetilde{x} = -\widehat{x}$ and $a_1 = 1/2$. Using this in (lxxxvi) one obtains

$$\widetilde{x} = \sqrt{\frac{\sigma^2}{2\lambda}\frac{c}{2(\lambda F + c)}\frac{1-H^2}{H} + \frac{\sigma^2}{4(\lambda F + c)}}. \tag{xcii}$$

If $\widetilde{x} = 0$, the DM may strictly prefer stopping search without adopting the alternative to deferring choice. So, the value matching condition $V(\widetilde{x}) - F = \widetilde{x}$ needs to be replaced by $V(\widetilde{x}) - F \geq \widetilde{x}$. In that case, $\widetilde{x}$ will be 0 rather than $\sqrt{\frac{\sigma^2}{2\lambda}\frac{c}{2(\lambda F+c)}\frac{1-H^2}{H} + \frac{\sigma^2}{4(\lambda F+c)}}$. Therefore, in general,

$$\widetilde{x} = \max\left\{\sqrt{\frac{\sigma^2}{2\lambda}\frac{c}{2(\lambda F + c)}\frac{1-H^2}{H} + \frac{\sigma^2}{4(\lambda F + c)}}, 0\right\} \tag{xciii}$$

Proof of Proposition 8:

The derivations for the comparative statics with regard to F and the comparative statics of the extent of choice closure with regard to c are straightforward.

According to (27) and $\widetilde{x} = -\widehat{x}$, if $\widetilde{x} > 0$, then

$$\text{sign}\left\{\frac{\partial(\widetilde{x} - \widehat{x})}{\partial c}\right\} = \text{sign}\left\{\frac{8Fc^2}{2c + \lambda F} - \sigma^2\right\} \tag{xciv}$$

First consider $\frac{8Fc^2}{2c+\lambda F} - \sigma^2 < 0$. Since $\widetilde{x} > 0$ is equivalent to $\sqrt{\frac{\sigma^2}{2\lambda}\frac{c}{2(\lambda F+c)}\frac{1-H^2}{H} + \frac{\sigma^2}{4(\lambda F+c)}} > 0 \Leftrightarrow 8F(2c + \lambda F) - \sigma^2 < 0 \Leftrightarrow c < \frac{\sigma^2}{16F} - \frac{\lambda F}{2}$, we have $\frac{\partial(\widetilde{x} - \widehat{x})}{\partial c} < 0$ if $c < \frac{\sigma^2}{16F} - \frac{\lambda F}{2}$ and $\frac{8Fc^2}{2c+\lambda F} - \sigma^2 < 0$, according to (xciv). $\widetilde{x} = 0$ and thus $\frac{\partial(\widetilde{x} - \widehat{x})}{\partial c} = 0$ if $c \geq \frac{\sigma^2}{16F} - \frac{\lambda F}{2}$ and $\frac{8Fc^2}{2c+\lambda F} - \sigma^2 < 0$.

Now consider $\frac{8Fc^2}{2c+\lambda F} - \sigma^2 \geq 0$. We have shown in the previous case that $\widetilde{x} = 0$ is equivalent to $8F(2c + \lambda F) - \sigma^2 \geq 0$. Since $8F(2c+\lambda F) - \sigma^2 > \frac{8Fc^2}{2c+\lambda F} - \sigma^2 \geq 0$, $\widetilde{x}$ is always 0 when $\frac{8Fc^2}{2c+\lambda F} - \sigma^2 > 0$.

In sum, the extent of choice closure always (weakly) decreases in c.

Now let's look at the comparative statics with regard to λ. One can see that H is increasing in λ. Therefore, $\frac{1-H^2}{H} = \frac{1}{H} - H$ is decreasing in λ and $\widetilde{x} - \widehat{x} = 2\widetilde{x} = \sqrt{\frac{\sigma^2}{2\lambda}\frac{c}{(\lambda F+c)}\frac{1-H^2}{H} + \frac{\sigma^2}{2(\lambda F+c)}}$ is decreasing in λ. So, the extent of choice deferral decreases in λ.

$$\begin{aligned}(28) \Rightarrow & \overline{x} - \widetilde{x} = \frac{1}{\widehat{\eta}}\ln H = \sqrt{\frac{\sigma^2}{2\lambda}}\ln\left[1 + \frac{\lambda F}{c} + \sqrt{\left(\frac{\lambda F}{c} + 1\right)^2 - 1}\right] \\ \Rightarrow & \text{sign}\left\{\frac{\partial(\overline{x} - \widetilde{x})}{\partial \lambda}\right\} = \text{sign}\left\{-\ln H + \frac{2\lambda F}{c\sqrt{\left(\frac{\lambda F}{c} + 1\right)^2 - 1}}\right\} = \text{sign}[G(\lambda)],\end{aligned} \tag{xcv}$$

where $G(\lambda) := -\ln H + \frac{2\lambda F}{c\sqrt{(\frac{\lambda F}{c}+1)^2 - 1}}$. One can see that $G(0) = 0$ and $G'(\lambda)$ is proportional to

$-(\frac{\lambda F}{c})^2 < 0$. Therefore, $G(\lambda) < 0, \forall \lambda > 0$. We then have that (xcv) implies that $\frac{\partial(\overline{x}-\widetilde{x})}{\partial \lambda} < 0$. So, the extent of choice closure decreases in λ.

Non-Random Switching Between Search and No-Search Modes:

In this section we study the DM's optimal search strategy under non-random switching between search states numerically. The length of time that the DM spends in each mode is deterministic. Specifically, the DM moves from the search mode to the no-search mode after $1/\lambda$ units of time and moves back from the no-search mode to the search mode after $1/\beta$ units of time.

We approximate continuous time using a discrete-time grid with increments of $dt = 0.001$. We simulate the stochastic movement of x using a discrete-time approximation to Brownian motion (Dixit 1993).

Because the DM knows whether she is able to search or not in the next "period", there is only one relevant adoption threshold at each t. Figure A.3 illustrates the decision thresholds under non-random switching. The DM's optimal search strategy is no longer stationary. The adoption threshold in the search mode, $\overline{x}$, decreases in time during a search period, so the DM is more likely to adopt the alternative the longer she searches within a session. The extent of choice deferral, $\widetilde{x}$, increases in time during the no-search period. When the DM recovers from the no-search mode to the search model, the decision threshold resets. One can interpret the difference between $\overline{x}$ at the beginning of the search mode and the $\widetilde{x}$ at the beginning of the no-search mode as the extent of choice closure.

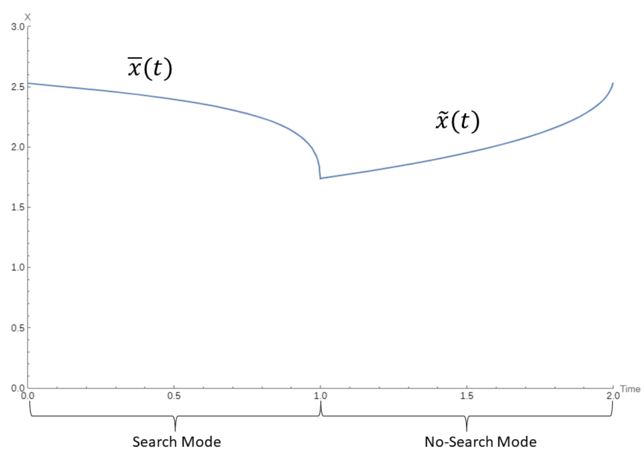

Figure A.3: Example of the adoption threshold as a function of time for $r = .05$, $\lambda = 1$, $\beta = 1$, and $\sigma = 1$.

For a fixed t, we also observe that $\overline{x}$ and $\widetilde{x}$ move in the same directions as they do under the base model when β, λ, r, and σ^2 change.

Search Strategy Under Deadline:

In this section we study the case with a decision deadline numerically. If the DM has not adopted the alternative after time T, then the decision becomes obsolete, and the DM receives a utility of 0.

We approximate continuous time using a discrete-time grid with increments of $dt = 0.001$. We simulate the stochastic movement of x using a discrete-time approximation to Brownian motion.

Figure A.4 illustrates the decision thresholds. The DM's optimal search strategy is no longer stationary. Both the extent of deferral, $\widetilde{x}$, and the extent of closure, $\overline{x} - \widetilde{x}$, decrease in time and approach 0 as $t \to T$. The DM is more likely to adopt the alternative over time.

For a fixed t, we observe that $\overline{x}$, $\widetilde{x}$, and $\overline{x} - \widetilde{x}$ move in the same directions as they do under the base model when β, λ, r, and σ^2 change.

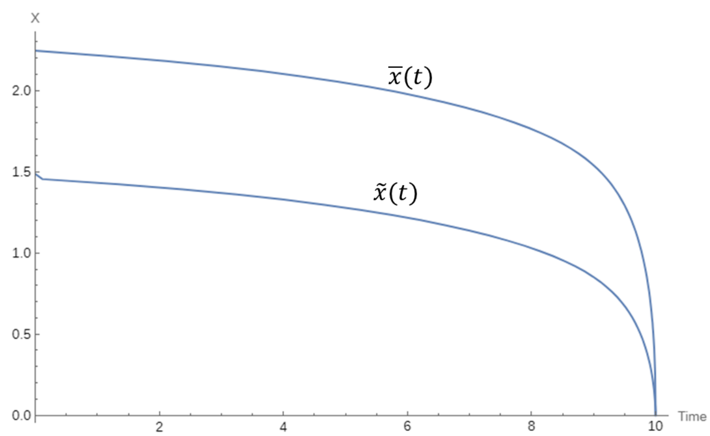

Figure A.4: Example of the adoption thresholds as a function of time for $r = .05$, $\lambda = 1$, $\beta = 1$, $\sigma = 1$, and $T = 10$.