Reputation for Privacy

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Abstract

As consumers become increasingly concerned about their privacy, previous literature has shown that firms can benefit from committing not to sell consumer data. However, the holdup problem prevents them from doing so in a static setting. This paper studies whether the reputation consideration of the firm can serve as a commitment device in a long-run game when consumers have imperfect monitoring technology. A patient enough monopoly can commit because its reputation will be permanently destroyed if consumers observe the data sale. The persistent punishment provides the monopoly a strong incentive not to deviate. In contrast, reputation may fail to serve as a commitment device when there are multiple firms. The penalty for selling data is smaller when consumers cannot know exactly which firm sold the data. Also, other firms can hurt the reputation of a particular firm even if that firm does not sell data. This paper finds sufficient conditions under which the incentive to deviate is so strong that duopolistic firms lose the ability to commit even if they are arbitrarily close to perfectly patient. Furthermore, reputation consideration of the firm cannot serve as a commitment device for privacy under any conditions when there are many firms. Reputation failure in the presence of many firms persists with endogenous or asymmetric monitoring.

1   Introduction

An information market has emerged in the digital era. The business of collecting and selling consumer data is estimated to be worth around \$200 billion.[1] Firms use detailed information about individuals to offer a personalized product, price discriminate, show targeted ads, etc. Aware of the costs of revealing information, consumers are becoming increasingly concerned about their privacy. People started to raise concerns about their privacy in the 1990s. About .01% of the US population opted out of the database of Lotus MarketPlace.[2] But most people at that time were either not aware of privacy issues or did not care much about them. By contrast, a 2021 survey of the general US population by KPMG found that 86% of consumers viewed data privacy as a growing concern.[3]

One reason people worry about a firm collecting their data is that they do not know how the firm will use it. According to the same survey, 40% of consumers do not trust firms to use their data ethically. Taylor (2004) shows that a firm can be better off by not protecting consumer privacy if consumers are naive and unaware that the firm sells their data. However, selling data can backfire if consumers are sophisticated and expect the firm to sell their data. A large body of literature has documented that the ability to commit to consumer privacy benefits the firm. As a result, companies pay increasing attention to privacy. Apple, for instance, has invested heavily in operating systems to protect consumer privacy and spent a good deal on advertising its progress in privacy protection. In this particular setting, the firm desires to commit to protecting consumer privacy by not selling consumer data. However, the non-verifiable nature of digital data makes it hard for a firm to commit. Despite the emerging regulations about consumer privacy, such as GDPR and CCPA, there are concerns about the credibility of such policies. Even if firms do not sell data in the presence of such regulation, consumers cannot easily verify it. Protecting consumer privacy will not benefit

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[1] https://www.latimes.com/business/story/2019-11-05/column-data-brokers

[2] https://www.forbes.com/sites/forbestechcouncil/2020/12/14/the-rising-concern-around-consumer-data-and-privacy/?sh=73c76330487e

[3] https://advisory.kpmg.us/content/dam/advisory/en/pdfs/2021/corporate-data-responsibility-bridging-the-consumer-trust-gap.pdf

the firm if it fails to obtain consumers' trust in how it handles their data. This paper looks at one possible solution - building trust by reputation.

The main contribution of the paper is to characterize conditions such that reputation considerations can or cannot serve as a commitment device for privacy. When a monopoly is patient enough, reputation enables it to commit never to sell data. It achieves the Stackelberg payoff in all but a finite number of periods. However, when there are multiple firms, reputation may fail to enable any firms to commit. We find sufficient conditions under which the incentive to deviate is so strong that duopolistic firms lose the ability to commit even if they are arbitrarily close to perfectly patient. Furthermore, reputation consideration of the firm cannot serve as a commitment device for privacy under any conditions when there are many firms. Such reputation failure hurts all the firms.

We consider long-lived firms interacting with short-lived consumers repeatedly in two markets. In the product market, the consumer decides how much information to reveal. Each firm infers consumer preferences based on the revealed information and offers a personalized product and price. The consumer then makes the purchase decision. By revealing more information, they get a better recommendation.[4] However, the firm will charge a higher price when it collects more information from the consumer, knowing that they have a higher expected valuation for the product. So, the consumer faces a tradeoff between better product fit and lower price. In the information market, the firm can sell consumer data to third parties (e.g., data intermediaries). Consumers may suffer disutility from the sale of their data. For example, they may experience scam emails/calls or account hacking. If consumers reveal more information, they will be more vulnerable to data sales. Therefore, the consumer's decision as to how much information to reveal in the product market depends on their belief about the firm's behavior in the information market. If the consumer thinks the firm will sell their data, they will reveal no information, to minimize privacy loss. If they trust the firm not to sell their data, they will reveal some information, to get a better product recommendation.

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[4] We refer to the consumer as the inclusive singular "they" throughout the paper.

The Stackelberg action of the firm is not to sell data. But whether to sell data or not is decided after the consumer reveals the information. Hence, the holdup problem prevents the firm from not selling data in a static setting.

This paper studies whether the reputation consideration of the firm can serve as a commitment device in a long-run game when consumers have imperfect monitoring technology. Reputation can be a commitment device for a patient enough monopoly but may fail to be one when there are multiple firms, even when firms are arbitrarily patient. The intuition is that a monopoly's reputation depends solely on its own actions. The monopoly will never restore its reputation by deviating from selling the data and being caught. The high and permanent reputation cost strongly incentivizes the monopoly to commit to privacy. In contrast, when there are multiple firms, consumers do not know exactly which firm sold the data, even if they observe data sales. One firm's reputation depends on other firms' actions. Selling data by one firm imposes a negative externality on other firms. Firms do not take this into account in equilibrium. The benefit of not selling data is lower because other firms' behavior may still hurt the firm's reputation. Anticipating this externality, consumers penalize each firm less when observing data sales. In addition, the likelihood that the deviation is pivotal decreases in the number of firms. Therefore, the cost of selling data is lower, and a firm's reputation may be hurt even if it does not sell data. In this case, the firm has more incentive to deviate.

We consider several extensions to the main model. Consumers can voluntarily exert efforts to better monitor firms. Endogenous monitoring helps a monopoly build up a reputation faster, benefiting both the rational firm and consumers. However, it does not provide enough incentives for multiple firms to commit not to sell data. Also, we consider asymmetric monitoring. The monopoly case implies that rational firms can commit if the monitoring technology is perfect. In contrast, any noise will break down the commitment power. This fragility means that the *possibility* of interaction of firms' behavior in the reputation-building process - rather than the *level* of interaction - is critical to reputation failure.

This paper contributes to the literature on the economics of privacy (see Acquisti, Taylor, and Wagman 2016 for a survey). Goldfarb and Tucker (2012) and Lin (2022) document the existence of substantial consumer privacy concerns. In a static framework, Ichihashi (2020) shows that sellers prefer to commit to the price of the good so that buyers will reveal more information. We investigate when such commitment is feasible without an external commitment device. Recent papers have paid much attention to the economic impact of regulations such as GDPR, CCPA, and AdChoices, which seek to protect consumers' privacy and give them more control over their data (Goldfarb and Tucker 2011, Conitzer, Taylor, and Wagman 2012, Campbell, Goldfarb, and Tucker 2015, Athey, Catalini, and Tucker 2017, Gardete and Bart 2018, Anderson, Baik, and Larson 2019, Goldberg, Johnson, and Shriver 2019, Montes, Sand-Zantman, and Valletti 2019, Choi, Jerath, and Sarvary 2020, 2023, Johnson, Shriver, and Du 2020, Rafieian and Yoganarasimhan 2021, Choi and Jerath 2022, Hu, Momot, and Wang 2022, Ke and Sudhir 2022, Lei, Miao, and Momot 2022, Ning, Shin, and Yu 2022, Bonatti, Huang, and Villas-Boas 2023, Fainmesser, Galeotti, Momot 2023, Johnson, Shriver, and Goldberg 2023, Bondi, Omid, and Yao 2024).

There are two reasons why reputation is essential despite various regulations. First, the main focus of those regulations is to give consumers more control over their data usage, rather than to provide the firm with commitment power. Second, the opacity and non-verifiability of data transactions raise concerns about the credibility of such policies. Even if firms do not sell data in the presence of such regulation, consumers still may not reveal enough information to the firm. Protecting consumer privacy will not benefit the firm if it fails to obtain consumers' trust in how firms handle their data. Absent an information market and the possibility of selling data, Chen and Iyer (2002) study competing firms' incentives to collect data. They find that firms may voluntarily collect less information about consumers to mitigate price competition. Closely related to our paper, Jullien, Lefouili, and Riordan (2020) study a website's incentive to sell consumer information in a two-period model. Unlike our setup, the website in their paper does not try to change consumers' beliefs about its type.

Instead, the website wants to affect consumer behavior based on their vulnerability and bad experiences due to data sales.

This paper is also related to the reputation literature. The idea of modeling reputation by incomplete information comes from Kreps et al. (1982), Kreps and Wilson (1982), and Milgrom and Roberts (1982). Fudenberg and Levine (1989) show that a patient long-run player will commit to the Stackelberg action in the presence of a behavioral type and perfect monitoring. Reputation serves as a commitment device and selects away bad equilibria for the long-lived player. In contrast, a strand of literature on bad reputation, including Ely and Välimäki (2003) and Morris (2001), shows that reputation concerns may hurt the firm under imperfect monitoring. Different from our papers where a firm can build reputation by always taking the Stackelberg action (optimal action with commitment power) in the absence of other firms, reputation failure in the bad reputation literature relies on a player's inability to build reputation by always taking the Stackelberg action. Substantively, the paper most closely related to ours is Phelan (2006), which studies a problem where the government builds a reputation for trust. Despite perfect monitoring, the reputation shock is non-permanent because the government's type can change over time.

Tirole (1996) initiates the study of collective reputation, where individual reputation and incentive depend on both one's own and other players' past behavior. Unlike our paper which looks at the entire equilibrium, Tirole (1996) focuses on the steady state and enforces bad behavior in the initial period. Neeman, Öry, and Yu (2019) compares collective reputation and individual reputation. They characterize conditions such that socially optimal actions are more likely to be sustained under collective reputation rather than individual reputation. Collective reputation is also studied in the umbrella branding literature (Wernerfelt 1988, Cabral 2000, Kuksov 2007, Miklós-Thal 2012, Moorthy 2012, Kuksov, Shachar, and Wang 2013, Zhang 2015, Klein et al. 2019, Yu 2021, Ke, Shin, and Yu 2023). Despite the long development of the reputation literature, researchers have not paid much attention to the reputation for privacy. Our paper contributes to the reputation literature by investigating

the economic and managerial implications of reputation for privacy, and by connecting bad reputation and collective reputation. The underlying mechanism that drives reputation failure is a combination of bad reputation and collective reputation, and is qualitatively different from either the bad reputation literature or the collective reputation literature.

Lastly, by building its reputation for privacy, rational firms wishes to separate them from other firms who does not care about consumer privacy. So, our paper is also related to the literature on signaling (Gal-Or, Geylani, and Dukes 2008, Iyer and Kuksov 2010, Kuksov and Wang 2013, Miklós-Thal and Zhang 2013, Jiang, Ni, and Srinivasan 2014, Dai and Singh 2020, Cao, Chen, and Ke 2023, Yang, Shi, and Lin 2023, Ke et al. 2024).

The remainder of the paper is organized as follows. Section 2 presents the main model. Section 3 presents the equilibrium of a static game. Section 4 characterizes the equilibrium of the repeated game when there is a monopoly. Section 5 solves the equilibrium of the repeated game when there are multiple firms. Section 6 considers two extensions to the main model. Section 7 concludes.

2 Model

Time is infinite, $t = 0, 1, 2, ...$, and the discount factor is $\delta < 1$. There are N long-lived firms and a short-lived consumer in each period. The consumer interacts with all the firms. The firm's payoff is $(1 - \delta) \sum_{t=0}^{+\infty} \delta^t u_t$, where u_t is the stage payoff at time t. There is a product market and an information market.

2.1 Product Market

Consumers have different horizontal preferences and are located uniformly on a circle with a circumference of 1. When a consumer visits a firm in the product market, they choose how much information to reveal. If a consumer located at $x \sim [0, 1)$ reveals $\eta \in [0, 1]$ proportion of information, the firm gets a noisy signal $l \sim U[x - (1-\eta)/2 \mod 1, x + (1-\eta)/2$

mod 1] about the consumer's location, as illustrated by Figure 1. This implies that the firm will offer a product located at l given the signal. The firm offers a personalized product and sets the price p based on the signal. The consumer then makes the purchase decision. Denote the distance between the product's location, y, and the consumer's location, x, by $d = |x - y|$. We have that $d \sim U[0, (1 - \eta)/2]$. The baseline valuation of the product is v, and the disutility from the mismatch of the recommended product and the consumer's horizontal taste is td. Therefore, the consumer gets $v - td - p$ if they buy and 0 if they do not buy. Different firms offer different products, so the consumer may buy from multiple firms as long as the product offered by each firm gives the consumer a positive expected payoff. We assume that there is enough horizontal differentiation, $t > v$.

The setting of the firm offering one personalized product and price to each consumer based on the information the consumer reveals is analogous to the setting in Ichihashi (2020). The implicit assumption that each firm offers only one product and a consumer cannot purchase products not recommended by the firm can be justified by the consumer's limited attention. We acknowledge that the modeling choice does not fully capture the real-world details. However, product market is not the focus of this paper, and this simple setup can concisely reflect the consumer's trade-off between better recommendation and higher price due to the hold-up problem by revealing more information, which is the main economic forces in the product market and will be discussed in detail in section 3.1.
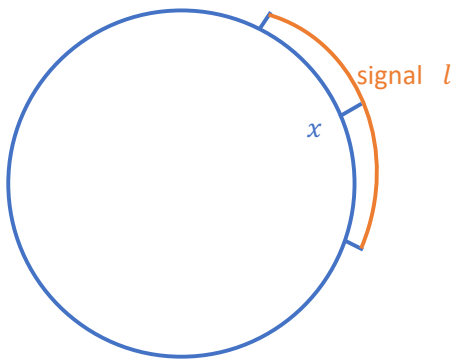

Figure 1: Consumer's location x and the signal l

2.2 Information Market

A firm can sell consumer data to third parties (e.g., data intermediaries) in the information market. It has the data directly revealed by the consumer, as well as the consumer's behavioral data (whether they purchase given the product and price offered). It can infer consumers' willingness to pay from the data they reveal directly (Bergemann, Bonatti, and Gan 2022), or from their behavioral data (Shen and Villas-Boas 2018, Taylor 2004, Villas-Boas 1999, 2004). A firm gets $D(\eta)$ by selling data. We assume that $D(\eta)$ increases in η to reflect that more accurate information is more valuable.

Consumers might experience a scam or account hack if a firm sells data. They are more vulnerable to such undesired activities when they reveal more information. So, we assume that the expected privacy cost of the consumer is ηu_b.[5] Consumers can imperfectly monitor firms' behavior in the information market. If a firm sold their data, the consumer detects data sales with probability q.[6] The consumer receives a signal $s = y$ if they caught any of the sales and $s = n$ if they did not detect any sales. The assumption that consumers cannot distinguish which firm sold the data gives the sharpest illustration of the main idea. In section 6.2, we relax the assumption by allowing consumers to have a better sense of which firm sold the data.

2.3 Reputation

We follow the standard way of modeling reputation by incompete information, originated from a series of papers by Kreps et al. (1982), Kreps and Roberts (1982), and Milgrom and Roberts (1982). There are two types of firms. A rational type (type R) maximizes the expected sum of discounted utilities, whereas a behavioral type (type B) always sells consumer data. One can view the behavioral type as a myopic firm who only cares about the present. An alternative and better interpretation is not to think about the behavioral type literally.

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[5] The main property we need is that the consumer's expected privacy loss is larger when the consumer reveals more information. We choose this particular form for simplicity.

[6] This simple informaton structure is analogous to the setting in Tirole (1996).

Rather, it is a standard modeling device in the reputation literature that introduces incomplete information in order to model reputation. Our focus is on the equilibrium behavior of the rational firm. The firm's reputation is consumers' belief about the probability that the firm is type B. The common prior that each firm is type B is $\mu_0 \in (0,1)$. Consumers update their belief about the firm's type by Bayes' rule. Denote the belief about firm i's type at time t by $\mu_{i,t}$. Reputation for privacy in this paper refers to the reputation for protecting consumer privacy in the information market. The consumer's current belief is a measurement of the firm's reputation.

2.4 Timing

Figure 2 illustrates the timing of the game.

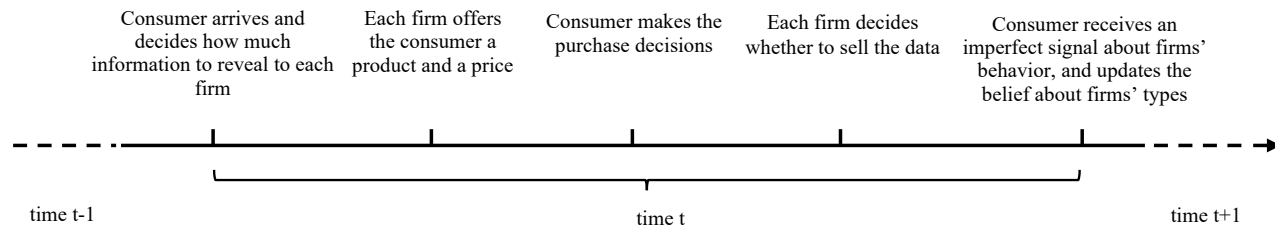| Consumer arrives and decides how much information to reveal to each firm | Each firm offers the consumer a product and a price | Consumer makes the purchase decisions | Each firm decides whether to sell the data | Consumer receives an imperfect signal about firms' behavior, and updates the belief about firms' types |

time t-1   time t   time t+1

Figure 2: Timing of the Game

2.5 Solution Concept

We focus on whether a Markov Perfect Equilibrium (MPE) exists where rational firms could commit to never selling the data.[7] MPE requires that firms' and consumers' strategies depend only on the current state. It is widely used in the reputation literature. The belief $\vec{\mu}_t = (\mu_{1,t}, \mu_{2,t}, ..., \mu_{N,t})$ is a natural state variable. In the symmetric equilibrium where every firm always has the same reputation, we use $\mu_t = \mu_{i,t}$ as the state variable. Importantly, and

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[7] Technically, the solution concept we use is the Markov Perfect Bayesian Equilibrium, because there is incomplete information about the firm's type. However, the reputation literature usually uses the notion of MPE.

consistent with the majority of literature on reputation, a player's strategy in each period is a function of the current belief rather than the whole history.

3   Equilibrium in a Static Game With One Firm

We first analyze the equilibrium outcome of a static game with one period and a single firm to prepare us for the analyses of the entire game.

3.1   Product Market

Given the product recommendation and price, the consumer purchases if and only if the expected payoff is positive, $v - td - p \geq 0$. Thus, the firm's problem is:

$$\max_{p} p \cdot \mathbb{P}[v - td - p \geq 0] = p \cdot \min\left\{\frac{2(v - p)}{(1 - \eta)t}, 1\right\}$$

Therefore, the optimal price is:

$$p^*(\eta) = \begin{cases} \frac{v}{2}, & if\ \eta \leq 1 - \frac{v}{t} \\ v - \frac{(1 - \eta)t}{2}, & if\ \eta > 1 - \frac{v}{t} \end{cases}$$

When the consumer reveals a lot of information, $\eta > 1 - \frac{v}{t}$, the firm accurately knows their preference. The recommended product is always close to the consumer's actual location, and the firm can extract a high surplus, even from the consumer located farthest away from the recommended product. Therefore, the firm sets a price such that the consumer always purchases. When the consumer reveals less information, the firm gets a noisier signal about their preference. The profit from each purchase will be too low if the firm wants the consumer to always buy the product. Therefore, only consumers with a high enough valuation for the recommended product purchase it at the optimal price. If the recommended product is located too far away the consumer, the consumer will not buy it.

From the consumer's perspective, revealing more information has two opposite consequences in the product market. On one hand, the firm can offer a better-matched product, which benefits the consumer. On the other hand, the firm will charge a higher price, knowing that the consumer has a higher expected valuation, which hurts the consumer.[8] In an extreme case, if the firm knows the consumer's preference perfectly, it will extract all the consumer surplus. Therefore, the consumer never reveals everything. In the other extreme case, if the firm knows nothing about the consumer's preference, it can only recommend a random product. The poor match also hurts the consumer. So, it is optimal for the consumer to reveal partial information if they only consider the product market.

3.2   Information Market

The consumer also needs to consider the effect of information revelation in the information market. Disclosing more information to the firm always hurts the consumer there, as they are more vulnerable when the firm sells their data. The probability that a firm sells data is endogenously determined in equilibrium. Given a probability μ_s, the consumer's ex-ante expected payoff is:

$$U_0(\eta) = \begin{cases} -\mu_s \eta u_b + \frac{v^2}{4(1-\eta)t}, & if \ \eta \leq 1 - \frac{v}{t} \\ -\mu_s \eta u_b + \frac{(1-\eta)t}{4}, & if \ \eta > 1 - \frac{v}{t} \end{cases}$$

The first term of the payoff, $-\mu_s \eta u_b$, is the consumer's expected privacy loss. Conditional on the firm selling data, the consumer's expected privacy cost is ηu_b. Given a probability μ_s of selling data, the consumer's ex-ante expected privacy loss is $-\mu_s \eta u_b$. The second term of the payoff, $\frac{v^2}{4(1-\eta)t}$, $if \ \eta \leq 1 - \frac{v}{t}$ and $\frac{(1-\eta)t}{4}$, $if \ \eta > 1 - \frac{v}{t}$, is the consumer's expected payoff in the product market. It is always positive because the consumer purchases the product only if the expected payoff is positive.

Considering both the product and information markets, it is never optimal for the con-

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[8] For more discussion about this kind of holdup problem, see Villas-Boas (2009) and Wernerfelt (1994).
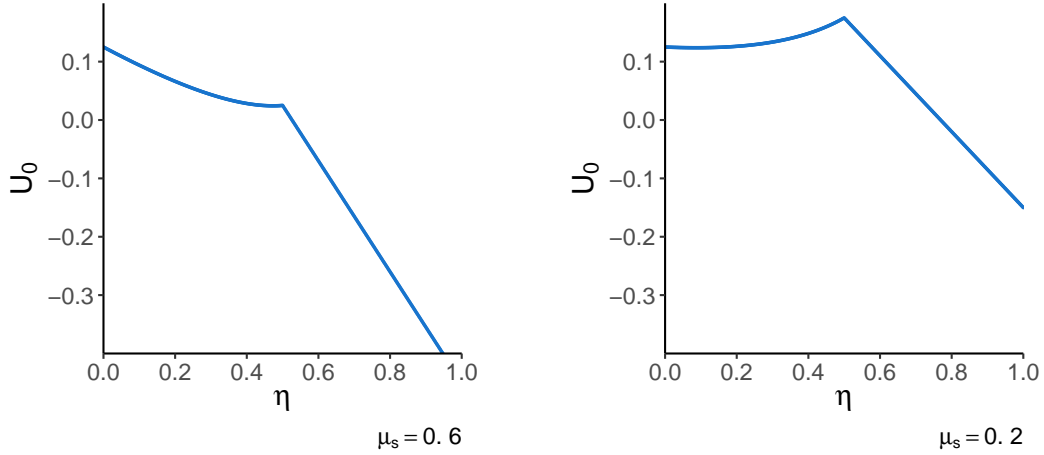
Figure 3: Ex-ante consumer payoff as a function of η for $v = 1$, $t = 2$, $u_b = .75$, and $\mu_s = .6$ (left) or .2 (right).

sumer to reveal too much information, as Figure 3 illustrates. The firm can charge a high price because the product recommendation is pretty accurate, with lots of information about the consumer. In addition, the privacy loss from data sales in the information market is high. Consumers may, however, prefer revealing a moderate amount of information to revealing nothing. By revealing some information, consumers benefit from a better recommendation in the product market but suffer a privacy cost if the firm sells it in the information market. If the firm's likelihood of selling the data is high, the high expected privacy loss in the information market outweighs the gain from the better match in the product market. In that case, the consumer reveals no information. On the contrary, the consumer partially reveals their preference for a better recommendation if the firm's likelihood of selling data is low. The following result formalizes our intuition. For the problem to be interesting, we assume that the threshold $\widehat{\mu} \in (0, 1)$. We also assume that they choose $\eta = 1 - v/t$ when they are indifferent between $\eta = 1 - v/t$ and 0, which does not affect any analyses.

Proposition 1. *The consumer does not reveal any information if $\mu_s > \widehat{\mu}$, and reveals $\eta^* = 1 - v/t$ amount of information if $\mu_s \leq \widehat{\mu}$, where $\widehat{\mu} = \frac{v}{4u_b}$.*

Corollary 1. *The firm's profit in the product market is* $\Pi^* = \begin{cases} v/2, & if \ \mu_s \leq \widehat{\mu} \\ v^2/2t, & if \ \mu_s > \widehat{\mu} \end{cases}$.

If the firm could commit not to sell consumer data, the consumer would choose $\eta = 1 - v/t$, which gives the firm a stage payoff of $v/2$. If, instead, the firm always sells consumer data, the consumer will not reveal any information (i.e., they will choose $\eta = 0$). In that case, the firm obtains a stage payoff of $v^2/2t + D(0)$.

Denote by $\Delta u := v/2 - v^2/2t - D(0)$ the difference between the firm's expected stage payoff with and without commitment. The firm will always sell consumer data if $\Delta u < 0$ and the entire problem becomes trivial. So, we focus on the interesting case in which $\Delta u > 0$ throughout the remaining analyses.

3.3   Some Benchmarks

We now analyze the property of the stage game of a single firm by comparing with some benchmarks. If the firm can commit to any action in the information market (e.g., by moving first), the firm takes the *Stackelberg action* and obtains the *Stackelberg payoff*.

Definition 1. *Suppose player 1 chooses action $a_1 \in A$ and player 2 chooses action $b \in A_2$. Player $i \in \{1, 2\}$'s stage-game payoff is $u_i(a, b)$. $BR_2(a) \subset A_2$ is player 2's best response correspondence to a. Then, player 1's Stackelberg action is* $\arg \max_{a_1 \in A_1} [\min_{a_2 \in BR_2(a)} u_1(a, b)]$, *and player 1's Stackelberg payoff is* $\max_{a_1 \in A_1} [\min_{a_2 \in BR_2(a)} u_1(a, b)]$.

One can see that the Stackelberg action for the firm is not to sell the data. The consumer will reveal $\eta = 1 - v/t$ proportion of information, and the firm gets the Stackelberg payoff of $v/2$.

If the consumer acts first and minimizes the firm's payoff, the firm gets the *minmax payoff*, which is the payoff the firm can guarantee regardless of the consumer's action.

Definition 2. *Suppose player 1 chooses action $a_1 \in A_1$ and player 2 chooses action $a_2 \in$*

A_2. *Player $i \in \{1, 2\}$'s stage-game payoff is $u_i(a, b)$. Then, player 1's minmax payoff is*

$$\min_{\beta \in \Delta(A_2)} [\max_{a_1 \in A_1} u_1(a, b)].$$

To minimize the firm's payoff, one can see that the consumer will reveal no information. In response, the firm sells data and gets the minmax payoff of $v^2/2t + D(0)$.

In a static game, the firm always sells data because it decides whether to sell data after the consumer reveals information. Anticipating this, the consumer reveals nothing. The firm receives the minmax payoff. Our objective in this paper is to study whether reputation considerations in the dynamic game enable the firm to commit to the Stackelberg action and receive the Stackelberg payoff.

4 Monopoly

4.1 Belief Updating

We first derive the consumer's belief updating processes about a monopoly's type, assuming that the rational type never sells the data. We can derive the belief updating when the rational firm uses other strategies by similar methods. Consider the consumer's belief about the monopoly's type after observing a signal s. If $s = y$, the consumer knows for sure that the firm sold the data in the previous period. So, the belief that the firm is a bad type will be one forever. The firm suffers a permanent reputation shock. If $s = n$, either the firm is the rational type and did not sell the data, or the firm is the bad type but the consumer fails to detect data sales. The firm is more likely to be the rational type. So, the consumer's belief that firm 1 is a bad type decreases but is still positive. Formally, the belief updating is as follows.

Proposition 2. *Suppose the rational type never sells the data in equilibrium.* $\mu_{t+1} =$
$$\begin{cases} \frac{1-q}{1-q\mu_t}\mu_t, & \text{if } s = n \\ 1, & \text{if } s = y \end{cases}$$ *. After receiving signal n for k consecutive periods, the belief becomes*

$\mu_{t+k} = \frac{(1-q)^k}{(1-q)^k \mu_t + 1 - \mu_t} \mu_t$, *which approaches 0 as* $k \to +\infty$.

So, if the monopoly continues not to sell consumer data, the consumer's belief will keep decreasing. After enough time, the consumer is almost certain that the firm is not the bad type.

4.2 Equilibrium

Suppose consumers expect the rational firm never to sell the data in equilibrium. In that case, a signal n will destroy the firm's reputation by making the consumer believe that the firm is the bad type in all current and future periods. Then, the firm is stuck with the minmax payoff. By deviating, the firm risks being detected by the consumer with a positive probability. The persistent punishment strongly incentivizes the firm not to sell the data for short-term benefit. As a result, regardless of the monitoring technology or the price of data in the information market, reputation can always serve as a commitment device as long as the monopoly is patient enough.

Proposition 3. *There exists a* $\widehat{\delta} < 1$ *such that, for any* $\delta > \widehat{\delta}$, *there exists a MPE where the rational firm never sells consumer data and the consumer always reveals* $\eta = 1 - v/t$ *proportion of information after a finite period.*

By protecting consumer privacy, the rational firm keeps reducing the consumer's belief that it is a bad type. When the belief is below a threshold, the consumer is willing to reveal some information, which benefits the firm. A patient firm does not want to deviate, because the consumer may observe the deviation and believe that the firm is the bad type. If that happens, the consumer will never reveal any information. So, the firm permanently suffers from less revenue in the product market. This severe punishment provides a strong incentive for the firm to trade the short-term benefit of selling data in the information market for the long-term benefit of earning a higher profit in the product market.

5 Multiple Firms

When there is more than one firm, reputation may fail to serve as a commitment device for privacy. The difference comes from the interaction of firms' behavior in the reputation-building process.

5.1 Belief Updaing

When there are multiple firms ($N \geq 2$), the belief updating is qualitatively different from the monopoly case. Consider the consumer's belief about firm 1's type after observing a signal s, assuming that a rational firm never sells the data. If $s = y$, the consumer knows that at least one firm sold the data in the previous period but is unsure whether firm 1 sold it. So, the belief that firm 1 is a bad type increases but is still lower than 1, unlike the monopoly case. The reputation shock is temporary, and firm 1 can rebuild its reputation. Conditional on other firms' behavior, the likelihood that $s = n$ if firm 1 is a rational type and did not sell the data is higher than the case where firm 1 is a bad type, but consumers cannot directly observe the data sales. Consequently, firm 1 is more likely to be a rational type, but the consumer is uncertain. So, the belief that firm 1 is a bad type decreases but is still positive. Formally, the belief updating is as follows.

Proposition 4. *Suppose the rational type never sells the data in equilibrium. The posterior belief* $\mu_{t+1} = \begin{cases} \frac{1-q}{1-q\mu_t}\mu_t, & \text{if } s = n \\ \frac{1-(1-q)(1-q\mu_t)^{N-1}}{1-(1-q\mu_t)^N}\mu_t, & \text{if } s = y \end{cases}$ *, which does not depend on the number of firms* N *if* $s = n$ *and decreases in* N *if* $s = y$.

Even if firm 1 sold the data and the consumer observes a signal y, they know that at least one firm sold the data but does not know which firm. Therefore, they penalize firm 1 less than they do in the monopoly case. The reputation cost is temporary, and the consumer's belief will decrease if they receive signal n in the future. When there are more firms, the signal's noise is larger, and the consumer has less idea about which firm sold the data.
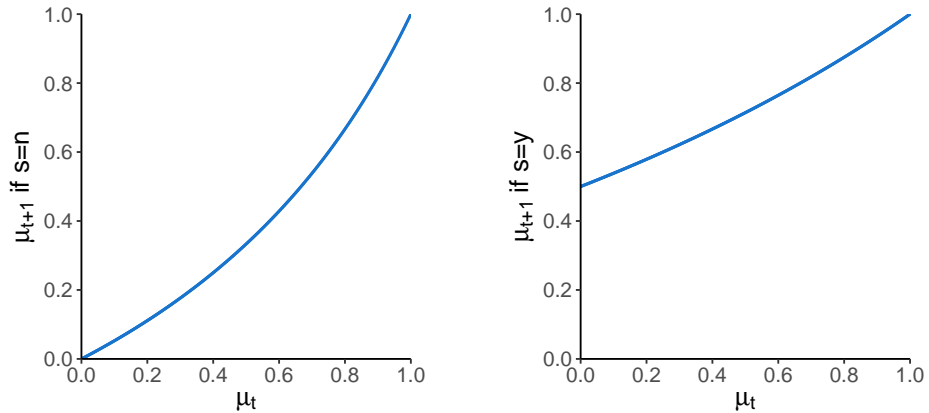
Figure 4: Belief updating as a function of μ_t for $q = .5$ and $N = 2$.

Therefore, there will be smaller increases in belief in response to signal y. If firm 1 did not sell the data and the consumer observes a signal n, the belief reduction does not depend on the number of firms. So, the firm is penalized less for selling the data but not rewarded more for not doing so. In addition, the realization of the signal depends little on a single firm's action when there are many firms. So, the likelihood that firm 1's action is pivotal decreases in the number of firms. Figure 4 illustrates the belief updating when there are two firms.

The above forces imply that the firm has more incentive to sell the data when the number of firms increases.

5.2 Two Firms

In this section, we study whether it is possible to achieve commitment by reputation when there are two firms.

Proposition 5. *Suppose there are two firms. There does not exist any MPE in which any rational firm could commit to never sell data, even when $\delta \to 1$, if the following conditions*

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hold:

$$\Delta u < \overline{\Delta u} := \frac{vq(t-v)}{2t} \tag{1}$$

$$q(1-q) < \frac{2D(0)}{v} \tag{2}$$

$$v/u_b < 2 \tag{3}$$

This proposition identifies sufficient conditions under which firms cannot commit even if they are arbitrarily close to perfectly patient. The role of each condition is the following.

○ Condition (1)

This intuitive condition means that the commitment payoff is not much higher than the one without commitment. The whole idea of building up reputation is to sacrifice short-term gains for long-term benefits. The left hand side of the condition, Δu, is a measure of long-term benefits. Proposition 3 has shown that reputation consideration provides a monopoly with a strong enough incentive to commit not to sell consumer data when there is any long-term benefit, $\Delta u > 0$. In contrast, this condition shows that, when there are two firms, reputation consideration cannot serve as a commitment device even if there is positive long-term benefit, as long as it is not too large, $\Delta u \in (0, \overline{\Delta u})$.

○ Condition (2)

This condition means that the noise of the monitoring technology is either low or high. Because of the imperfect monitoring technology, consumers may get a false signal y if a firm did not sell data or n if a firm sold data. If the signal's noise is high, the consumer is likely to get a signal y even if a firm did not sell data, because the other firm sold data. If the noise is low, the consumer is likely to get a signal n even if a firm sold data, because of poor monitoring. In either case, the likelihood that selling data is pivotal is low. This creates a stronger incentive for the firm to deviate.

○ Condition (3)

The consumer enjoys a positive utility from consuming good products in the product market. The valuation of the product, v, is related to the benefit of revealing some information to the firm. The consumer may be hurt if the firm sells their data. The cost of revealing a certain amount of information increases in u_B. This condition means that the consumer's benefit from revealing some information to the firm is not too high relative to its cost. A lower benefit to cost ratio leads to a lower cutoff belief of revealing information, $\widehat{\mu}$. In particular, a lower $\widehat{\mu}$ implies that a good reputation is less persistent. Even if the firm does not sell data and reduces the belief below the threshold, signal y in future periods will more easily raise the belief above $\widehat{\mu}$, inducing the consumer to stop revealing any information. The fast depreciation of reputation makes building it less attractive.

When all these three conditions hold, reputation considerations provide no commitment power to rational firms.,[9]

5.3 Many Firms

In the previous section, we have characterized a set of conditions under which reputation consideration of the firm fails to serve as a commitment device for privacy even if duopolistic firms are arbitrarily close to perfectly patient. In this section, we show that it is even harder for each firm to commit not to sell consumer data when there are more firms. In particular, the next result shows that reputation consideration of the firm cannot serve as a commitment device for privacy *under any conditions* when there are many firms.

Proposition 6. *For any $\delta \in (0, 1), \exists N_\delta$ s.t. $\forall N \geq N_\delta$, firms always sell data, and consumers reveal nothing in the unique MPE. Consumers' belief about each firm's type is always μ_0.*

No matter how patient firms are, they cannot build a reputation for privacy. The intuition for the failure of reputation as a commitment device is the following. On one hand, the

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[9] For example, these conditions will hold if the privacy cost for consumers is high and the monitoring technology has low noise.

consumer has a noisy signal about which firm sold the data. Even if the firm deviates and the consumer observes it, the penalty for that particular firm is less than that for the monopoly firm. Moreover, the penalty decreases in the number of firms. On the other hand, even if none of rational firms deviates, other firms may be the bad type and sell the data. So, as the number of firms increases, it becomes less likely that an individual firm's sale of data is pivotal. Both forces give the firm more incentive to deviate and sell the data. So, it becomes harder to commit when the number of firms increases. Eventually, the firm loses all the commitment power and sells data every period.

Anticipating that the firm will always sell data, consumers do not reveal anything. The belief of each firm's type remains the same over time, and there is no reputation building. The monopoly can get Stackelberg payoffs in all but a finite number of periods under substantial punishment for selling data. In contrast, each firm can only get the minmax payoff under weaker punishment when there are multiple firms, even if there is no competition.

5.4   Relationship With Previous Work

Prisoners' Dilemma

One may think the result that a monopoly can commit but more firms cannot commit even though they would be collectively better off with commitment is similar to prisoners' dilemma. However, there are key difference between our insights and mechanisms and those in prisoners' dilemma.

Prisoner's dilemma considers a static game. The key economic force for each player to deviate from cooperation is the lack of future rewards or punishments. Coorperation can happen when we consider a repeated version of the game because non-cooperation has long-term impact on the player. Our findings are all based on repeated interactions. The result that no firm can commit to never sell consumer data even if they are arbitrarily close to perfectly patient and there are an infinite number of periods is qualitatively different

from prisoners' dilemma. Our mechanism is a collection of bad reputation and collective reputation, which has not been jointly studied to our knowledge.

Bad Reputation

In the bad reputation literature, initiated by Morris (2001) and Ely and Välimäki (2003), a rational player's incentive to avoid bad reputation eliminates all the welfare because the player may need to distort the action from the Pareto-optimal one to the action that hurts everyone's short-term payoffs due to reputation considerations. The reputation failure results appear even if there is only one long-run player. By contrast, a patient enough firm can build the reputation and achieves the Stackelberg payoff in the absence of other long-lived firms and collective reputation.

The key difference is that the firm's reputation may be hurt even if it takes the optimal action for the consumer in the bad reputation literature, whereas a monopoly always improves its reputation by doing the right thing for the consumer in our paper. Fundamentally, the difference comes from the different market structures. Usually, in the bad reputation literature, there is a single market and the bad type's incentive is partially misaligned with the consumer. The partial misalignment implies that the bad type's action may be sometime optimal for the consumer. Consequently, in order to separate it from the bad type, the rational/good type may need to take an action that hurts the consumer (and the firm). In contrast, the unique feature about privacy is that there are two markets. The firm and the consumer's incentive are partially misaligned in the product market: the firm always benefits from more information while the consumer wants to reveal some information but not everything if we only consider the product market. In the information market, however, the bad type's incentive is completely misaligned with the consumer: the firm wants to sell the data while the consumer does not. As a result, the rational type can always separate it from the bad type by taking the optimal action for the consumer in the information market. Therefore, the presence of multiple firms and collective reputation is critical to the reputation

failure results.

5.5 Managerial Implications

Even though commitment may be desirable for the firm, it may not be possible without strict external regulations. A monopoly can always build a reputation for caring about consumer privacy by not selling data. After a finite period, consumers will reward it by sharing more information. The monopoly can enjoy a high profit by recommending better-fit products and charging a premium. However, when there are multiple firms in the market, it may not be in the firm's best interest to protect consumer privacy. Even if a firm never sells consumer data, it may not be able to build a reputation for privacy. So, it loses the revenue from selling consumer information, while does not have any (or enough) gain. Since firms benefit from committing never to sell consumer data, they need to think about other ways of achieving the commitment. Our model shows that the key to commitment power is the tradeoff between the short-term benefit in the information market and the long-term benefit in the product market. A potential solution is to improve the recommendation algorithm so that the firm has a higher marginal benefit from consumer information. It will have a stronger incentive to maintain a good reputation in order to profit from the product market. The other solution is to invest in better monitoring technology to make it easier for consumers to identify which firm sells data. Lastly, the firm can compensate consumers if the signal is y. In that case, the firm will face an additional penalty for selling consumer data. Therefore, the "free lunch" in the information market is more costly for the firm. This idea is related to restoring truthful communication between an advisor and a decision maker by monetary incentives, studied in Durbin and Iyer (2009).

In reality, firms are actively lobbying in favor or against government regulations. Another implication of our results is that, in markets where firms earn most of their revenue from the actual products rather than advertising (in our model, we do not consider advertising revenue, which depends crucially on the amount of information about individual ocnsumers

a firm can collect), oligopoly firms should support privacy regulations more than monopoly ones.

6 Extensions

6.1 Endogenous Monitoring

The monitoring technology is exogenous in the main model. In reality, consumers observe some data sales without any effort. They know their phone number has been sold if they get a scam call. If they get a pre-approved credit card with their name in the mail, they know that some firms have sold their address and credit history. However, as consumers become more concerned about privacy issues, they may endogenously invest in better monitoring of firms' use of their data; they can do this by exerting more effort, such as purchasing a security app. In this section, we consider this possibility and allow for endogenous monitoring.[10]

The setup is the same as before, except that the consumer can incur costs to obtain an extra signal s' about the data sale after observing the costless signal s. By exerting an effort $h \in [0, \bar{h}]$ ($\bar{h} < 1$), the consumer obtains a signal s_h. If a firm sold data in the previous period, the consumer detects it with probability h. The consumer would receive a signal $s_h = y$ if they caught any sales and $s_h = n$ if they did not detect any sales. We make the following assumption on the cost $c(h)$.

Assumption 1. $c(\cdot) \in \mathcal{C}^2(\mathbb{R}_+), c(0) = 0, c'(h) > 0, c''(h) > 0, \lim_{h \to \bar{h}} c'(h) = +\infty.$

We assume that it is costless for the consumer to exert zero effort, that the marginal monitoring cost increases at an increasing rate when the precision of monitoring improves, and that it is very costly to monitor data sales with a high degree of precision.

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[10] In a different setting, Singh (2017) shows that endogenous monitoring may or may not eliminate the corrupted behavior of the agent. The agent's corrupted behavior of selecting a nondeserving firm in exchange for bribes in their paper can be mapped to firms selling consumer data in our paper. That paper also makes a similar assumption on the monitoring technology and cost.

6.1.1 Monopoly

Let μ be the consumer's belief after observing signal s. The consumer can incur effort h to gain an additional signal s_h. We consider the MPE in which the rational firm never sells data. Suppose there exists such an equilibrium. There are two cases.

i. $\mu > \widehat{\mu}$: Without an extra signal, the consumer will reveal nothing, according to Proposition 1. If the consumer obtains a costly signal, they must take a different action under some circumstances. Otherwise, they will be better off by not incurring any costs. Therefore, the belief must be below $\widehat{\mu}$ if the consumer exerts effort h and receives a signal $s_h = n$. When the belief μ is too high, the updated belief will be above $\widehat{\mu}$ regardless of the effort. Therefore, the consumer will not incur a cost to get an extra signal. When the belief μ is close to $\widehat{\mu}$, the belief will be below $\widehat{\mu}$ if the consumer exerts enough effort and receives signal n. The consumer benefits from costly monitoring by being more likely to identify the rational type.

ii. $\mu \leq \widehat{\mu}$: Without an extra signal, the consumer will reveal $1 - \eta/t$ amount of information according to Proposition 1. If the consumer seeks an extra signal and receives $s_h = y$, they know that the firm is a bad type and does not reveal information. If $s_h = n$, the belief decreases, and the consumer reveals some information. The consumer benefits from costly monitoring by being more likely to identify the bad type.

Examining both cases, we have the following result.

Proposition 7. *There exists a $\widehat{\delta} < 1$, such that for any $\delta > \widehat{\delta}$, there exists a MPE where the rational firm never sells consumer data. In such equilibrium, there exists a $\widehat{\widehat{\mu}} > \widehat{\mu}$, such that the consumer exerts efforts in monitoring if and only if $\mu \leq \widehat{\widehat{\mu}}$. The monitoring effort strictly increases in μ for $\mu > \widehat{\mu}$. It vanishes as μ approaches zero.*

Figure 5 illustrates the optimal monitoring effort as a function of the belief. As we can see, the consumer does not incur any monitoring costs when the belief is far above the threshold belief of revealing information, $\widehat{\mu}$. When the consumer strongly believes that
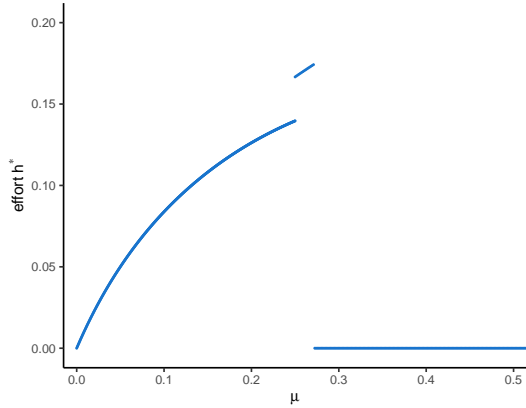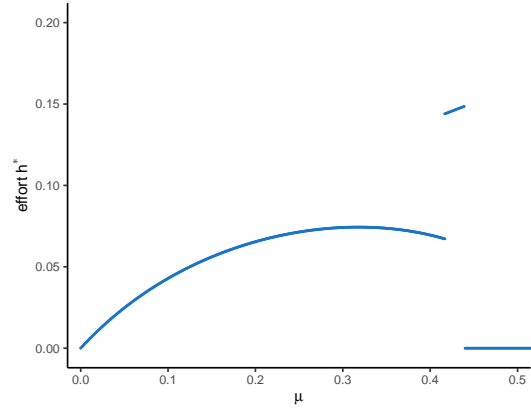
(a) $v = 1, t = 3, u_b = 1, \bar{h} = .5, k = .1$ (b) $v = 1, t = 3, u_b = .6, \bar{h} = .5, k = 1$

Figure 5: The optimal monitoring effort, where $c(h) = \frac{kh^2}{\bar{h}-h}$.

the firm is rational, they also incur little cost because the likelihood of detecting the data sale is very low. They will reveal the same amount of information without an extra signal. Hence, costly monitoring provides little benefit to her. In contrast, additional monitoring will be valuable for the consumer when the belief is slightly above $\widehat{\mu}$. Since the consumer is quite uncertain about the firm's type, their expected payoff is low. If they reveal nothing and the firm is a rational type, they give up the opportunity of receiving better product recommendations. If they reveal some information and the firm is bad, they suffer a high privacy loss. By getting another signal, the consumer becomes more certain about the firm's type. A y signal convinces the consumer that the firm will sell their data. So, they reveal nothing. An n signal makes them more confident that the firm will not sell their data. So, they reveal some information. Consequently, the consumer exerts a relatively high effort in this case.

Consumers start revealing information at a higher belief when they can voluntarily monitor the firm's behavior. So, the rational firm builds up its reputation and achieves the Stackelberg payoff faster under endogenous monitoring. The consumer makes better decisions with an extra signal. Both players are better off.

6.1.2 Multiple Firms

Under exogenous monitoring and multiple firms, we have the reputation failure results. Under endogenous monitoring and a monopoly, the consumer's ability to gain additional signals makes it easier for the firm to build its reputation. One natural question is whether endogenous monitoring suffices to restore reputation when there are multiple firms. The following result shows that it is not enough to reverse the negative results.

Proposition 8. *For any $\alpha > 0$ and $\delta \in (0,1), \exists N_\delta$ s.t. $\forall N \geq N_\delta$, firms always sell data and consumers reveal nothing in the unique MPE. Consumers' belief about each firm's type is always μ_0.*

Even though endogenous monitoring can help firms build up their reputation faster when they do not sell data, consumers are more likely to detect data sales, which hurts firms' reputation. With the possibility of a bad type who always sells data, rational firms are tempted to sell data as well, because their reputation is affected by other firms. When the number of firms increases, it becomes harder for rational types to commit. Eventually, the firm loses all its commitment power and sells data every period.

6.2 Asymmetric Monitoring

In the main model, the monitoring technology of the consumer is symmetric. The consumer observes whether some firms sold the data without further information about which firm is more likely to sell it. We now consider an asymmetric monitoring technology where consumers have a better sense of whether a particular firm sold the data.

If firm 1 sold the data in the previous period, the consumer detects it with probability q. If firm $i \neq 1$ sold the data in the previous period, the consumer detects it with probability αq ($0 < \alpha < 1$). The consumer would receive a signal $s = y$ if they caught any sales and $s = n$ if they did not detect any. The consumer faces less noise about whether firm 1 sold the data. To get some intuition, notice that $\alpha = 1$ corresponds to the monitoring technology

in the main model. The consumer cannot distinguish at all which firm sold the data. If $\alpha = 0$ instead, the consumer knows for sure that firm 1 sold the data in the previous period upon receiving signal $s = y$. The result in the monopoly case, Proposition 3, implies that a sufficiently patient firm 1 can commit to never selling data, no matter how large the total number of firms is. When α is between 0 and 1, the consumer knows after observing signal $s = y$ that firm 1 is more likely to be the firm that sold the data in the previous period, but there is some noise. It may have been firm 2 (or any other firm other than firm 1) that sold the data, though with a lower likelihood. When α is close to 1, there is a lot of noise, and each firm may have sold the data with similar probabilities. When α is close to zero, the monitoring technology is close to the monopoly case. Firm 1 is much more likely to have deviated than firm 2. The monitoring technology is close to the monopoly case. The following result shows that any noise from other firms will break down the commitment power and lead to reputation failure.

Proposition 9. *For any $\alpha > 0$ and $\delta \in (0, 1), \exists N_\delta$ s.t. $\forall N \geq N_\delta$, firms always sell data and consumers reveal nothing in the unique MPE. Consumers' belief about each firm's type is always μ_0.*

The fragility demonstrated by this result means that the *possibility* of interaction of firms' behavior in the reputation-building process, rather than the *level* of interaction, is critical to reputation failure.

7 Conclusion

This paper studies whether reputation consideration can serve as a commitment device for privacy. We show that it depends on the market structure. For a patient enough monopoly, reputation enables it to commit to the Stackelberg action of not selling consumers' data. This is because, when consumers observe that data has been sold, they know that the monopoly

sold it. So, the firm will never restore its reputation if it is caught selling data. The high and permanent reputation cost strongly incentivizes the monopoly to commit to privacy.

In contrast, reputation may fail to help the firm commit not to sell data when there are multiple firms. We characterize some sufficient conditions under which firms cannot commit even if they are arbitrarily close to perfect patient. In this case, consumers can never know exactly which firm sold their data. Therefore, the penalty for data sales is lower. However, a firm's reputation may be hurt even if it does not sell data. The minor and temporary reputation cost strongly incentivizes the firm to deviate.

Reputation failure in the presence of multiple firms persists when we consider several extensions. Endogenous monitoring helps a monopoly build up a reputation faster, benefiting both the rational firm and consumers. However, it does not provide enough incentives for multiple firms to commit not to sell data. Also, we consider asymmetric monitoring. The monopoly case implies that rational firms can commit without noise. In contrast, any noise from other firms will break down the commitment power. This result shows that the *possibility* rather than the *level* of interaction is critical to reputation failure.

There are a couple of limitations to the current work. Consumers can reveal an arbitrary amount of information in the product market. However, a firm sometimes restricts the communication space. So, the consumer can only choose from a menu of the amount of information to disclose. It will be interesting to study the optimal design of the menu and how much advantage a firm could gain by offering such a contract. Also, the consumer's privacy loss from data sales is exogenous in this paper. Endonenizing the privacy cost in a game theoretic model can provide further insights. We leave these topics for future research.

Appendix

Proof of Proposition 1. The consumer's expected ex-ante payoff by choosing to reveal η proportion of information is:

$$U_0(\eta) = \begin{cases} -\mu_s \eta u_b + \frac{v^2}{4(1-\eta)t}, \ if \ \eta \leq 1 - \frac{v}{t} \\ \\ -\mu_s \eta u_b + \frac{(1-\eta)t}{4}, \ if \ \eta > 1 - \frac{v}{t} \end{cases}$$

$U_0(\eta)$ decreases in η for $\eta > 1 - \frac{v}{t}$, so the consumer will not reveal more than $1 - \frac{v}{t}$ proportion of information. Consider $\eta \in [0, 1 - \frac{v}{t}]$.

$$\frac{dU_0(\eta)}{d\eta} = -\mu_s u_b + \frac{v^2}{4t(1-\eta)^2}$$

, which increases in η. So, the optimal η is either 0 or $1 - \frac{v}{t}$. $U_0(1 - \frac{v}{t}) \geq U_0(0) \Leftrightarrow \mu_s \leq \widehat{\mu}$, where $\widehat{\mu} = \frac{v}{4u_b}$. □

Proof of Proposition 2. Since the rational type does not sell the data, a signal $s = y$ implies the firm is the behavioral type. So, $\mu_{t+1} = 1$. Now consider $s = n$. By Baye's rule,

$$\begin{aligned} \mathbb{P}[type \ B|s = n] &= \frac{\mathbb{P}[s = n|type \ B]\mathbb{P}[type \ B]}{\mathbb{P}[s = n|type \ B]\mathbb{P}[type \ B] + \mathbb{P}[s = n|type \ R]\mathbb{P}[type \ R]} \\ &= \frac{(1-q)\mu_t}{(1-q)\mu_t + 1 \cdot (1 - \mu_t)} \\ &= \frac{1-q}{1 - q\mu_t}\mu_t \end{aligned}$$

By induction, we have $\mu_{t+1} = \frac{(1-q)\mu_t}{(1-q)\mu_t + 1 - \mu_t}$ after receiving signal n once. Suppose $\mu_{t+k} = \frac{(1-q)^k}{(1-q)^k\mu_t + 1 - \mu_t}\mu_t$ after receiving signal n for k consecutive periods. After receiving signal n for $k+1$ consecutive periods, we have $\mu_{t+k+1} = \frac{(1-q)\mu_{t+k}}{(1-q)\mu_{t+k} + 1 - \mu_{t+k}} = \frac{(1-q)^{k+1}}{(1-q)^{k+1}\mu_t + 1 - \mu_t}\mu_t$. So, it shows that $\mu_{t+k} = \frac{(1-q)^k}{(1-q)^k\mu_t + 1 - \mu_t}\mu_t$ after receiving signal n for k consecutive periods.

One can see that $\frac{(1-q)^k}{(1-q)^k\mu_t + 1 - \mu_t}\mu_t$ approaches 0 as $k \to +\infty$. □

Proof of Proposition 3. Let $\widehat{k} = \left\lceil \frac{\ln \frac{v(1-\mu_0)}{(4u_b-v)\mu_0}}{\ln(1-q)} \right\rceil$. We first show that the consumer reveals $\eta = 1 - v/t$ proportion of information after $\widehat{k}$ periods, if the firm never sells the data in equilibrium. By Proposition 2, the belief after not selling data for k consecutive periods is $\mu_k = \frac{(1-q)^k}{(1-q)^k\mu_0+1-\mu_0}\mu_0$. By Proposition 1, consumer reveals $\eta = 1 - v/t$ proportion of information if and only if $\mu_k \le \widehat{\mu} \Leftrightarrow k \ge \frac{\ln \frac{v(1-\mu_0)}{(4u_b-v)\mu_0}}{\ln(1-q)}$.

We now show that the rational firm has no incentive to deviate to selling data at any time. The game is continuous at infinity because of discounting. So, we can use the single-deviation property. Suppose the firm deviates once at period t when the belief is μ_t. There are two cases.

1. $\mu \le \widehat{\mu}$

   The value function of the equilibrium strategy (never sell data) is:

   $$V(\mu_t) = (1-\delta)\frac{v}{2}\frac{1}{1-\delta} = \frac{v}{2}$$

   The value function of deviating once at period t is (assuming the firm sells data when the belief is 1, which maximizes the payoff):

   $$\tilde{V}(\mu_t) = (1-\delta)[\frac{v}{2} + D(1 - \frac{v}{t}) + \delta(q\frac{\frac{v^2}{2t}+D(0)}{1-\delta} + (1-q)\frac{\frac{v}{2}}{1-\delta})]$$

   The rational firm will not deviate if $V(\mu_t) > \tilde{V}(\mu_t) \Leftrightarrow \frac{\delta}{1-\delta} > \frac{D(1-\frac{v}{t})}{q(v/2-v^2/2t-D(0))}$. One can see that $\exists \delta_1 \in (0,1)$ s.t. the inequality holds for any $\delta \ge \delta_1$.

2. $\mu > \widehat{\mu}$

   The value function of the equilibrium strategy (never sell data) is:

   $$V(\mu_t) = (1-\delta)[\sum_{k=0}^{\widehat{k}-1} \delta^k\frac{v^2}{2t} + \sum_{k=\widehat{k}}^{+\infty} \delta^k\frac{v}{2}]$$

   The value function of deviating once at period t is (assuming the firm sells data when the belief is 1, which maximizes the payoff):

   $$\tilde{V}(\mu_t) = (1-\delta)[\frac{v^2}{2t} + D(0) + \delta(q\frac{\frac{v^2}{2t}+D(0)}{1-\delta} + (1-q)[\sum_{k=0}^{\widehat{k}-2} \delta^k\frac{v^2}{2t} + \sum_{k=\widehat{k}-1}^{+\infty} \delta^k\frac{v}{2}])]$$

   The rational firm will not deviate if $V(\mu_t) > \tilde{V}(\mu_t) \Leftrightarrow \frac{\delta^{\widehat{k}}}{(1-\delta)[1-(1-q)\delta]} > \frac{D(0)}{q(v/2-v^2/2t)}$. One can see that $\exists \delta_2 \in (0,1)$ s.t. the inequality holds for any $\delta \ge \delta_2$.

Let $\widehat{\delta} = \max\{\delta_1, \delta_2\}$. One can see that $\widehat{\delta} < 1$ and for any $\delta > \widehat{\delta}$, the firm never sells consmer data, $\eta = 0$ in the first $\widehat{k}$ periods, and $\eta = 1 - v/t$ after $\widehat{k}$ periods is a MPE. $\square$

Proof of Proposition 4. By Baye's rule, for a given firm,

$$\mathbb{P}[type\ B|s = n]$$
$$= \frac{\mathbb{P}[s = n|type\ B]\mathbb{P}[type\ B]}{\mathbb{P}[s = n|type\ B]\mathbb{P}[type\ B] + \mathbb{P}[s = n|type\ R]\mathbb{P}[type\ R]}$$
$$= \frac{(1 - q)[1 \cdot (1 - \mu_t) + (1 - q)\mu_t]^{N-1}\mu_t}{(1 - q)[1 \cdot (1 - \mu_t) + (1 - q)\mu_t]^{N-1}\mu_t + 1 \cdot [1 \cdot (1 - \mu_t) + (1 - q)\mu_t]^{N-1}(1 - \mu_t)}$$
$$= \frac{1 - q}{1 - q\mu_t}\mu_t, \text{ which does not depend on } N.$$

$$\mathbb{P}[type\ B|s = y]$$
$$= \frac{\mathbb{P}[s = y|type\ B]\mathbb{P}[type\ B]}{\mathbb{P}[s = y|type\ B]\mathbb{P}[type\ B] + \mathbb{P}[s = y|type\ R]\mathbb{P}[type\ R]}$$
$$= \frac{\left[1 - (1 - q)[1 \cdot (1 - \mu_t) + (1 - q)\mu_t]^{N-1}\right]\mu_t}{\left[1 - (1 - q)[1 \cdot (1 - \mu_t) + (1 - q)\mu_t]^{N-1}\right]\mu_t + \left[1 - 1 \cdot [1 \cdot (1 - \mu_t) + (1 - q)\mu_t]^{N-1}\right](1 - \mu_t)}$$
$$= \frac{1 - (1 - q)(1 - q\mu_t)^{N-1}}{1 - (1 - q\mu_t)^N}\mu_t, \text{ which decreases in } N \text{ by checking the derivative.}$$

$\square$

Proof of Proposition 5. Consider firm 1 WLOG. We first list the updated belief after one signal:

$$\begin{cases} \mu^y = \mathbb{P}(\text{firm 1 is bad type}|s = y, \text{ initial belief is } \mu) = \frac{1 - (1-q)(1-q\mu)}{1 - (1-q\mu)^2}\mu \\ \mu^n = \mathbb{P}(\text{firm 1 is bad type}|s = n, \text{ initial belief is } \mu) = \frac{1-q}{1-q\mu}\mu \end{cases}$$

Both μ^y and μ^n increase in μ. $\mu^y \geq 1/2, \forall \mu$.

Suppose there exists an equilibrium in which rational firms never sells the data. Then consumers have identical beliefs for both firms. Denote the corresponding value function by

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$V(\cdot)$. Consider a belief $\mu > \widehat{\mu}$.

$$V(\mu) = (1-\delta)\frac{v^2}{2t} + \delta\left[q\mu V(\mu^y) + (1-q\mu)V(\mu^n)\right]$$

The value function of deviating once in the current period is:

$$V_{dev}(\mu) = (1-\delta)\left(\frac{v^2}{2t} + D(0)\right) + \delta\left[q[1+(1-q)\mu]V(\mu^y) + (1-q)(1-q\mu)V(\mu^n)\right]$$

$$V_{dev}(\mu) - V(\mu) = (1-\delta)D(0) - \delta\left[V(\mu^n) - V(\mu^y)\right]q(1-q\mu) \tag{4}$$

$v < 2u_b \Rightarrow \widehat{\mu} < 1/2$. $\mu^y \geq 1/2$, $\forall \mu$ implies that consumer will reveal no information after one signal y, which gives the rational firm a stage equilibrium payoff of $\frac{v^2}{2t}$. If the signal is n and $\mu^n \leq \widehat{\mu}$, rational firm gets a stage payoff of $v/2$; If the signal is n and $\mu^n > \widehat{\mu}$, rational firm gets a stage payoff of $\frac{v^2}{2t}$. So, we gets an upper bound of $V(\mu^n)$ by assuming that the belief is always no greater than $\widehat{\mu}$:

$$V(\mu^n) \leq (1-\delta)\left[\frac{v}{2} + \sum_{k=1}^{+\infty}\delta^k[q\mu\frac{v^2}{2t} + (1-q\mu)\frac{v}{2}]\right]$$
$$= (1-\delta)\frac{v}{2} + \delta[q\mu\frac{v^2}{2t} + (1-q\mu)\frac{v}{2}]$$

Always selling consumer data gives a lower bound on the value function:

$$V(\mu^y) \geq (1-\delta)\sum_{k=0}^{+\infty}\delta^k[\frac{v^2}{2t} + D(0)] = \frac{v^2}{2t} + D(0)$$

Hence, we have:

$$V(\mu^n) - V(\mu^y) \leq (1-\delta)\frac{v}{2} + \delta[q\mu\frac{v^2}{2t} + (1-q\mu)\frac{v}{2}] - [\frac{v^2}{2t} + D(0)]$$

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Plug it back to (4), we have:

$$V_{dev}(\mu) - V(\mu)$$

$$\geq (1-\delta)[D(0) - \delta q(1-q\mu)\frac{v}{2}] - \delta q(1-q\mu)\left[\delta[q\mu\frac{v^2}{2t} + (1-q\mu)\frac{v}{2}] - [\frac{v^2}{2t} + D(0)]\right] \quad (5)$$

With a strictly positive probability, the signal will be y for k consecutive periods, $\forall k$. Denote the belief after k consecutive signal y by μ^{y^k}. One can see that $\mu^y \in (\mu, 1), \forall \mu \in (0,1)$. So, μ^{y^k} strictly increases in k and is bounded by 1. Thus, $\{\mu^{y^k}\}_{k=1}^{+\infty}$ has a limit. Denote the limit by $\mu^{y^{+\infty}}$. We have $(\mu^{y^{+\infty}})^y = \mu^{y^{+\infty}} \Rightarrow \mu^{y^{+\infty}} = 1$. So, μ^{y^k} could be arbitrarily close to 1 with a strictly positive probability. If $(1-q)(v/2 - v^2/2t) < D(0)$, for large enough δ and μ, we have $\delta[q\mu\frac{v^2}{2t} + (1-q\mu)\frac{v}{2}] - [\frac{v^2}{2t} + D(0)] < 0$. If $q(1-q)v/2 < D(0)$, for large enough δ and μ, we have $D(0) - \delta q(1-q\mu)\frac{v}{2} > 0$. Together, we get that $(1-\delta)[D(0) - \delta q(1-q\mu)\frac{v}{2}] - \delta q(1-q\mu)\left[\delta[q\mu\frac{v^2}{2t} + (1-q\mu)\frac{v}{2}] - [\frac{v^2}{2t} + D(0)]\right] > 0, \overset{(5)}{\Rightarrow} V_{dev}(\mu) - V(\mu) > 0$. Therefore, rational firm will sell the data when the belief is μ and the discount factor is high enough. A contradiction. $\qquad\square$

Proof of Proposition 6. Fix $\delta \in (0,1)$. Suppose $\forall N_\delta$, $\exists N \geq N_\delta$ s.t. there exists a MPE in which a rational firm (label it by firm 1 WLOG) does not sell the data at $t = 0$. Denote the equilibrium strategy of all the firms by σ and the value function of firm 1 by $V_1(\cdot)$. The prior belief is $\vec{\mu}_0 = (\mu_0, \mu_0, ..., \mu_0)$. Denote the posterior belief upon observing signal y (n) by $\vec{\mu}^y$ $(\vec{\mu}^n)$ when the initial belief is $\vec{\mu}$ and the equilibrium strategy is σ.

Suppose $\mu_0 > \widehat{\mu}$.

$$V_1(\vec{\mu_0}) = (1-\delta)\frac{v^2}{2t} + \delta\left[\mathbb{P}(s = y|\sigma)V_1(\vec{\mu}_0^y) + \mathbb{P}(s = n|\sigma)V_1(\vec{\mu}_0^n)\right]$$

, where $\begin{cases} \mathbb{P}(s = n|\sigma) \leq (1-q\mu_0)^{N-1} \\ \\ \mathbb{P}(s = y|\sigma) = 1 - \mathbb{P}(s = n|\sigma) \geq 1 - (1-q\mu_0)^{N-1} \end{cases}$

The upper bound of the probability of signal n, $\mathbb{P}(s = n|\sigma)$, is obtained when no rational

firm sells data under σ given belief $\vec{\mu}_0$. The value function of firm 1 if it deviates once in the first period (denote the strategy by σ') is:

$$V_{1,dev}(\vec{\mu}_0) = (1 - \delta)\left(\frac{v^2}{2t} + D(0)\right) + \delta\left[\mathbb{P}(s = y|\sigma')V_1(\vec{\mu}_0^y) + \mathbb{P}(s = n|\sigma')V_1(\vec{\mu}_0^n)\right]$$

, where
$$\begin{cases} \mathbb{P}(s = n|\sigma') = (1 - q)\mathbb{P}(s = n|\sigma) \\ \mathbb{P}(s = y|\sigma') = 1 - \mathbb{P}(s = n|\sigma') \end{cases}$$
Therefore, we have:

$$V_{1,dev}(\mu_0) - V_1(\mu_0) = (1 - \delta)D(0) - \delta\left[V_1(\mu_0^n) - V_1(\mu_0^y)\right]q\mathbb{P}(s = n|\sigma)$$

Since $V_1(\cdot) \in [\frac{v^2}{2t}, \frac{v}{2} + D(1 - \frac{v}{t})]$, $V_1(\mu_0^n) - V_1(\mu_0^y) \leq \frac{v}{2} + D(1 - \frac{v}{t}) - \frac{v^2}{2t}$, which is a constant. $\mathbb{P}(s = n|\sigma) \leq (1 - q\mu_0)^{N-1} \to 0$ $(N \to +\infty)$. Hence, given δ, $\exists N_\delta$ s.t. $\forall N \geq N_\delta$, $V_{1,dev}(\mu_0) - V_1(\mu_0) > 0$. But we assume that firm 1 does not sell the data at $t = 0$. A contradiction.

In sum, firms always sell data and consumers reveal nothing at $t = 0$ in equilibrium. Anticipating that, the consumer's posterior belief about each firm's type is μ_0. This repeats in every period. Therefore, for any $\delta \in (0, 1)$, $\exists N_\delta$ s.t. $\forall N \geq N_\delta$, firms always sell data and consumers reveal nothing in the unique MPE. Consumer's belief about the firm's type is always μ_0.

Suppose $\mu_0 \leq \widehat{\mu}$. The first period payoff will be $\frac{v}{2}$ in equilibrium and $\frac{v}{2} + D(1 - \frac{v}{t})$ if the firm deviates. All the remaining proof is the same as above. $\square$

Proof of Proposition 7. Suppose there exists such an equilibrium.

(1) $\mu \leq \widehat{\mu}$

Without any monitoring effort, the consumer does not get an additional signal. So, the consumer reveal $1 - v/t$ amount of information according to Proposition 1. The expected

consumer surplus is:

$$CS(0, \mu) := -\mu u_b(1 - v/t) + v/4$$

By incuring effort h, the consumer receives an extra signal s_h. The updated belief will be:

$$\begin{cases} \frac{1-h}{1-h\mu}\mu, & if \ s_h = n \text{ (with probability } 1 - \mu h) \\ \\ 1, & if \ s_h = y \text{(with probability } \mu h) \end{cases}$$

The expected consumer surplus is:[11]

$$CS(h, \mu) := -c(h) - \frac{1-h}{1-h\mu}\mu u_b(1 - \frac{v}{t})(1 - \mu h) + \frac{v^2}{4t}\mu h + \frac{v}{4}(1 - \mu h), \ h \in [0, \bar{h}]$$

The difference of the expected consumer surplus between incurring monitoring effort h and no effort is:

$$\Delta CS(h, \mu) := CS(h, \mu) - CS(0, \mu) = -c(h) + \mu\frac{v}{4t}(t-v)h\left[-1 + \frac{4u_b(1-\mu)}{v(1-\mu h)}\right], \ h \in [0, \bar{h}]$$

(6)

The consumer incurs a strictly monitoring effort if and only if $\Delta CS(h, \mu) > 0$ for some $h \in (0, \bar{h}]$. Notice that $\Delta CS(0, \mu) = 0$. So, a sufficient condition for the consumer to incur a strictly monitoring effort is $\frac{\partial \Delta CS(h,\mu)}{\partial h}|_{h=0} > 0$.

$$\frac{\partial \Delta CS(h, \mu)}{\partial h} = -c'(h) + \frac{\mu v(t-v)}{4t}\left[-1 + \frac{4u_b(1-\mu)}{v(1-\mu h)^2}\right]$$

$$\frac{\partial \Delta CS(h, \mu)}{\partial h}|_{h=0} > 0 \Leftrightarrow -1 + \frac{4u_b(1-\mu)}{v} > 0$$

$$\Leftrightarrow \mu < 1 - \widehat{\mu}$$

If $v < 2u_b$, $\widehat{\mu} < 1/2 \Rightarrow \mu \leq \widehat{\mu} < 1 - \widehat{\mu}, \forall \mu \leq \widehat{\mu}$. So, the consumer always incurs a strictly

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[11] Technically, the domin is $h \in (0, \bar{h}]$. But, one can check that the expression holds for $h = 0$ as well.

positive monitoring effort when $\mu < \widehat{\mu}$.

Equation (6) implies that $h^*(\mu) \to 0$ as $\mu \to 0$, since $\Delta CS(h^*(\mu), \mu) \geq 0$

(2) $\mu > \widehat{\mu}$

Without any monitoring effort, the consumer does not get an additional signal. So, the consumer reveal nothing according to Proposition 1. The expected consumer surplus is:

$$\widetilde{CS}(0, \mu) := v^2/4t$$

By incuring effort h, the consumer receives an extra signal s_h. The updated belief will be:

$$\begin{cases} \frac{1-h}{1-h\mu}\mu, & if \ s_h = n \ (\text{with probability } 1 - \mu h) \\ \\ 1, & if \ s_h = y(\text{with probability } \mu h) \end{cases}$$

If μ is high enough such that $\frac{1-\bar{h}}{1-\bar{h}\mu}\mu > \widehat{\mu}$, the consumer will not reveal anything regardless of the signal realization. So, there is no gain from an additional signal and the consumer will not acquire an extra signal. For the consumer to incur costly monitoring, they must reveal some information ($\eta = 1 - v/t$) if $s_h = n$.[12] The expected consumer surplus is the same as the first case:

$$\widetilde{CS}(h, \mu) = CS(h, \mu) = -c(h) - \frac{1-h}{1-h\mu}\mu u_b(1 - \frac{v}{t})(1 - \mu h) + \frac{v^2}{4t}\mu h + \frac{v}{4}(1 - \mu h), \ h \in (0, \bar{h}]$$

The difference of the expected consumer surplus between incurring monitoring effort h

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[12] This may be worse than revealing nothing for the consumer. But the latter is always dominated by not incurring monitoring costs.

and no effort is:

$$\Delta \widetilde{CS}(h,\mu) := \widetilde{CS}(h,\mu) - \widetilde{CS}(0,\mu) = -c(h) + (1-\mu h)\frac{v(t-v)}{4t}\left[1 - \frac{4u_b(1-h)\mu}{v(1-\mu h)}\right], \ h \in (0,\bar{h}]$$

(7)

The consumer incurs a strictly monitoring effort if and only if $\Delta\widetilde{CS}(h,\mu) > 0$ for some $h \in (0,\bar{h}]$.

$$\Delta\frac{\partial\widetilde{CS}(h,\mu)}{\partial h} = -c'(h) + \frac{\mu v(t-v)}{4t}\left(\frac{4u_b}{v} - 1\right)$$

(8)

Since $c'(0) = 0$, $c(\cdot)$ is convex, and $\lim_{h\to\bar{h}} c'(h) = +\infty$, we have $\max_{0<h\leq\bar{h}} \Delta\widetilde{CS}(h,\mu) = \Delta\widetilde{CS}(\widehat{h}(\mu),\mu)$, where $\widehat{h}(\mu)(> 0)$ is determined by the first order condition:

$$c'(\widehat{h}(\mu)) = \frac{\mu v(t-v)}{4t}\left(\frac{4u_b}{v} - 1\right)$$

(9)

The consumer's optimal effort is either 0 or $\widehat{h}(\mu)$. This leads to the following lemma.

Lemma 1. *Suppose the belief is $\mu > \widehat{\mu}$. The consumer incurs monitoring effort $\widehat{h}(\mu)$ if and only if $\Delta\widetilde{CS}(\widehat{h}(\mu),\mu) > 0$.*

The next lemma characterize the optimal effort of the consumer.

Lemma 2. *There exists a $\widehat{\widehat{\mu}} > \widehat{\mu}$ such that the consumer incurs efforts in monitoring if and only if $\mu \leq \widehat{\widehat{\mu}}$. For $\mu \in [\widehat{\mu}, \widehat{\widehat{\mu}}]$, the optimal effort $h^*(\mu)$ strictly increases in μ.*

Proof. We first show that the optimal effort follows a cutoff strategy. Suppose the consumer incurs monitoring effort $\widehat{h}(\mu_1) > 0$ when the belief is $\mu_1 > \widehat{\mu}$. $\Delta\widetilde{CS}(h)$ and $\widehat{h}$ depend on μ. Lemma 1 implies that $\Delta\widetilde{CS}(\widehat{h}(\mu_1),\mu_1) > 0$. $\forall\mu_2 \in (\widehat{\mu},\mu_1)$. Since $\Delta\widetilde{CS}(h,\mu)$ strictly decreases in μ, we have that $\Delta\widetilde{CS}(\widehat{h}(\mu_2),\mu_2) \geq \widetilde{CS}(\widehat{h}(\mu_1),\mu_2) > \Delta\widetilde{CS}(\widehat{h}(\mu_1),\mu_1) > 0$, where the first inequality is by the optimality of $\widehat{h}(\mu_2)$ for $\widetilde{CS}(h,\mu_2)$, $h \in (0,\bar{h}]$. Therefore, there exists a $\widehat{\widehat{\mu}} \geq \widehat{\mu}$ such that the consumer incurs efforts in monitor-

ing if and only if $\mu \leq \widehat{\mu}$. For $\mu \in [\widehat{\mu}, \widehat{\widehat{\mu}}]$, the optimal effort $h^*(\mu) = \widehat{h}(\mu)$. According to equation (9), $\widehat{h}(\mu)$ strictly increases in μ.

We now show that $\widehat{\widehat{\mu}} > \widehat{\mu}$. This can be shown by consider $\mu = \widehat{\mu} + \varepsilon, h = \sqrt{\varepsilon}$. Taking Taylor expansion in the expression for $\Delta \widetilde{CS}(h, \mu)$ and let $\varepsilon \to 0$ gives the result.

So, we finish the proof of the lemma. $\qquad\square$

Consumer optimality has been shown in the above analyses. Noticing that the benefit for not selling data and the penalty for selling data are higher under endogenous monitoring. One can see that the rational firm has no incentive to deviate when it is patient enough by similar arguments as the proof of Proposition 3.

$\qquad\square$

Proof of Proposition 8. The proof of Proposition 6 applies to this case as well. $\qquad\square$

Proof of Proposition 9. Fix $\delta \in (0, 1)$. Suppose $\forall N_\delta$, $\exists N \geq N_\delta$ s.t. there exists a MPE in which a rational firm j does not sell the data at $t = 0$. Denote the equilibrium strategy of all the firms by σ and the value function of firm i by $V_i(\cdot)$. The prior belief is $\vec{\mu}_0 = (\mu_0, \mu_0, ..., \mu_0)$. Denote the posterior belief upon observing signal y (n) by $\vec{\mu}^y$ $(\vec{\mu}^n)$ when the initial belief is $\vec{\mu}$ and the equilibrium strategy is σ.

Suppose $\mu_0 > \widehat{\mu}$. There are two possibilities:

(1) $j = 1$

$$V_1(\vec{\mu}_0) = (1 - \delta)\frac{v^2}{2t} + \delta\left[\mathbb{P}(s = y|\sigma)V_1(\vec{\mu}_0^y) + \mathbb{P}(s = n|\sigma)V_1(\vec{\mu}_0^n)\right]$$

, where $\begin{cases} \mathbb{P}(s = n|\sigma) \leq (1 - \alpha q\mu_0)^{N-1} \\ \\ \mathbb{P}(s = y|\sigma) = 1 - \mathbb{P}(s = n|\sigma) \geq 1 - (1 - \alpha q\mu_0)^{N-1} \end{cases}$

The upper bound of the probability of signal n, $\mathbb{P}(s = n|\sigma)$, is obtained when no rational

firm sells data under σ given belief $\vec{\mu}_0$. The value function of firm 1 if it deviates once in the first period (denote the strategy by σ') is:

$$V_{1,dev}(\vec{\mu}_0) = (1 - \delta) \left(\frac{v^2}{2t} + D(0) \right) + \delta \left[\mathbb{P}(s = y|\sigma') V_1(\vec{\mu}_0^y) + \mathbb{P}(s = n|\sigma') V_1(\vec{\mu}_0^n) \right]$$

, where $\begin{cases} \mathbb{P}(s = n|\sigma') = (1 - q)\mathbb{P}(s = n|\sigma) \\ \mathbb{P}(s = y|\sigma') = 1 - \mathbb{P}(s = n|\sigma') \end{cases}$ Therefore, we have:

$$V_{1,dev}(\mu_0) - V_1(\mu_0) = (1 - \delta)D(0) - \delta \left[V_1(\mu_0^n) - V_1(\mu_0^y) \right] q\mathbb{P}(s = n|\sigma)$$

Since $V_1(\cdot) \in [\frac{v^2}{2t}, \frac{v}{2} + D(1 - \frac{v}{t})]$, $V_1(\mu_0^n) - V_1(\mu_0^y) \leq \frac{v}{2} + D(1 - \frac{v}{t}) - \frac{v^2}{2t}$, which is a constant. $\mathbb{P}(s = n|\sigma) \leq (1 - \alpha q\mu_0)^{N-1} \to 0$ ($N \to +\infty$). Hence, given δ, $\exists N_\delta$ s.t. $\forall N \geq N_\delta$, $V_{1,dev}(\mu_0) - V_1(\mu_0) > 0$. But we assume that firm 1 does not sell the data at $t = 0$. A contradiction.

(2) $j \neq 1$

$$V_j(\vec{\mu}_0) = (1 - \delta)\frac{v^2}{2t} + \delta \left[\mathbb{P}(s = y|\sigma) V_j(\vec{\mu}_0^y) + \mathbb{P}(s = n|\sigma) V_j(\vec{\mu}_0^n) \right]$$

, where $\begin{cases} \mathbb{P}(s = n|\sigma) \leq (1 - q\mu_0)(1 - \alpha q\mu_0)^{N-2} \\ \\ \mathbb{P}(s = y|\sigma) = 1 - \mathbb{P}(s = n|\sigma) \geq 1 - (1 - q\mu_0)(1 - \alpha q\mu_0)^{N-2} \end{cases}$
The upper bound of the probability of signal n, $\mathbb{P}(s = n|\sigma)$, is obtained when no rational firm sells data under σ given belief $\vec{\mu}_0$. The value function of firm j if it deviates once in the first period (denote the strategy by σ') is:

$$V_{j,dev}(\vec{\mu}_0) = (1 - \delta) \left(\frac{v^2}{2t} + D(0) \right) + \delta \left[\mathbb{P}(s = y|\sigma') V_j(\vec{\mu}_0^y) + \mathbb{P}(s = n|\sigma') V_j(\vec{\mu}_0^n) \right]$$

$$\text{, where} \begin{cases} \mathbb{P}(s = n|\sigma') = (1 - \alpha q)\mathbb{P}(s = n|\sigma) \\ \\ \mathbb{P}(s = y|\sigma') = 1 - \mathbb{P}(s = n|\sigma') \end{cases} \quad \text{Therefore, we have:}$$

$$V_{j,dev}(\mu_0) - V_j(\mu_0) = (1 - \delta)D(0) - \delta \left[V_j(\mu_0^n) - V_j(\mu_0^y)\right] \alpha q \mathbb{P}(s = n|\sigma)$$

Since $V_j(\cdot) \in [\frac{v^2}{2t}, \frac{v}{2} + D(1 - \frac{v}{t})]$, $V_j(\mu_0^n) - V_j(\mu_0^y) \leq \frac{v}{2} + D(1 - \frac{v}{t}) - \frac{v^2}{2t}$, which is a constant. $\mathbb{P}(s = n|\sigma) \leq (1 - q\mu_0)(1 - \alpha q\mu_0)^{N-2} \to 0$ $(N \to +\infty)$. Hence, given δ, $\exists N_\delta$ s.t. $\forall N \geq N_\delta$, $V_{j,dev}(\mu_0) - V_j(\mu_0) > 0$. But we assume that firm j does not sell the data at $t = 0$. A contradiction.

In sum, firms always sell data and consumers reveal nothing at $t = 0$ in equilibrium. Anticipating that, the consumer's posterior belief about each firm's type is μ_0. This repeats in every period. Therefore, for any $\delta \in (0, 1), \exists N_\delta$ s.t. $\forall N \geq N_\delta$, firms always sell data and consumers reveal nothing in the unique MPE. Consumer's belief about the firm's type is always μ_0.

Suppose $\mu_0 \leq \hat{\mu}$. The first period payoff will be $\frac{v}{2}$ in equilibrium and $\frac{v}{2} + D(1 - \frac{v}{t})$ if the firm deviates. All the remaining proof is the same as above. $\square$

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